PHILADELPHIA UNIVERSITY
ELECTRICAL ENGINEERING DEPT.

POWER ELECTRONICS LECTURE NOTES

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LECTURE. 1: INTRODUCTION

1.1 Power Electronics Fundamentals

Power electronics relates power semiconductor devices circuitry, its design and role includes the techniques of converting and processing high power electrical energy. The role of power electronics is shown in Fig.1.

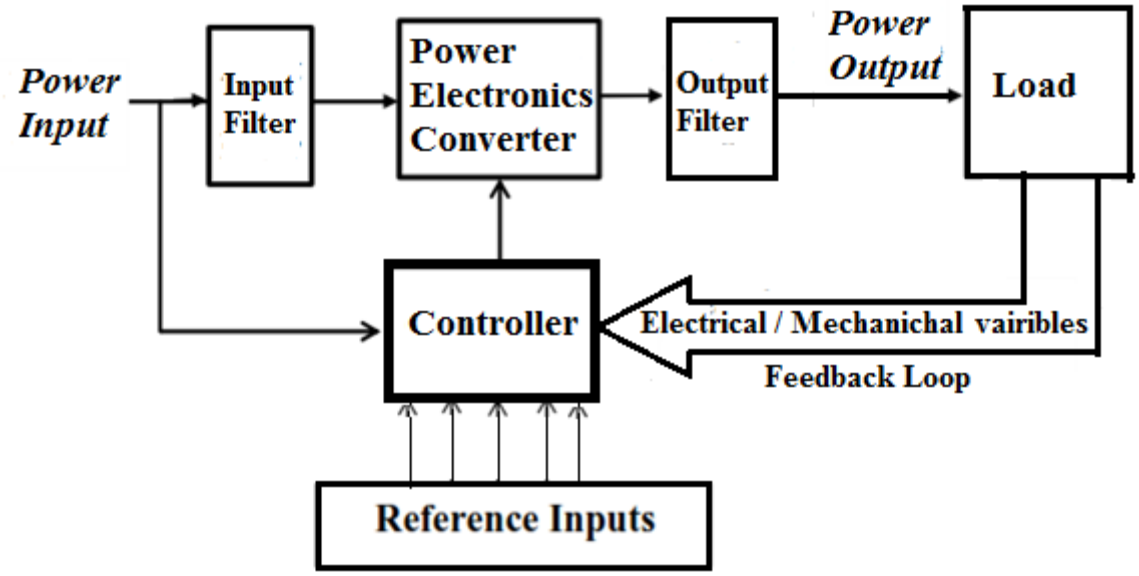


Fig. 1 The role of power electronics

Power semiconductor devices are used, such as power diodes, thyristors and power transistors. These power semiconductor devices are so important in modern power control .They are used in switching applications where they compete with “mechanically-operated" or “electrically-operated” switches and they give the following advantages:-

1. Faster in operation by a factor of at least a hundred times.
2. Need no routine maintenance.
3. Do not stick, bounce or wear.
4. Give no sparking on break.
5. Have a very long life expectancy when used properly .
6. Are physically small.
7. Are relatively cheap.

1.2.Goal of power electronic systems

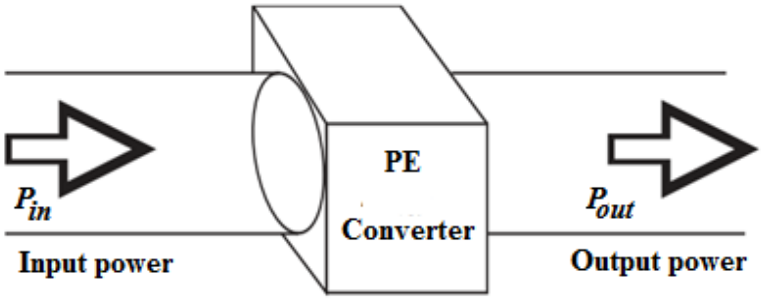
The main goal of electronic power conversion systems is to convert electrical energy from one form to another, from the source to load using power semiconductor device with:

»High efficiency
»High availability
»High reliability
»Small size
»Light weight and
»Low cost

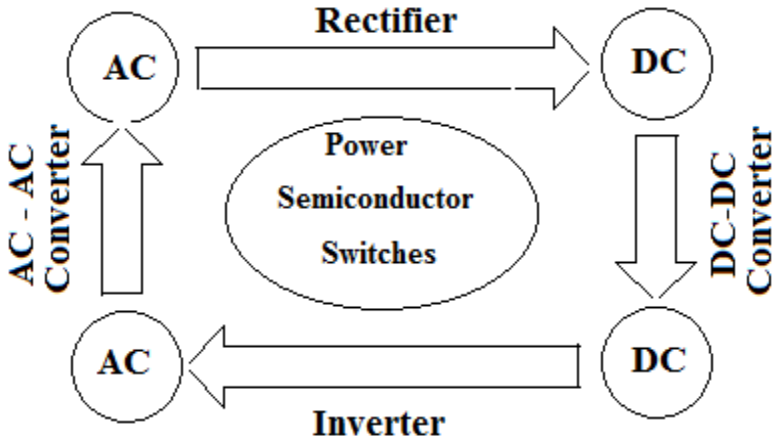
In addition to the simple on-off applications of power semiconductor devices their speed of operation and life expectancy, which are independent of the number of switching actions, make them possible to arrange for smooth control of the mean power from zero to maximum in most applications.

1.3 Power Electronics Applications

The main fields of applications are in Energy Processing and Drive Technology. Power semiconductor devices are interconnected in a variety of ways to form power control units, see Fig.2 (a). These units can be classified in a variety of ways, one of the simplest being to classify them according to their input/output requirements, thus, referring to Fig.1.2 (b).



(a)



(b)

Fig.1.2 Power or energy processing through power electronics converters.

(A) **Rectifiers** — circuits for changing ac voltage to dc voltage or transfer of power from an alternating current (ac) supply to direct current (dc) form. These circuits can be of two main types:

- AC - DC (uncontrolled)
- AC - DC (controlled)

(B) **DC - DC Converters** — Circuits which change a fixed d.c. voltage into a variable dc supply, i.e., transfer of power from a direct current supply directly into a direct current load of different voltage level. These types of converters are usually called DC choppers.

(C) **Inverters** — Circuits for changing a d.c. voltage to an alternating one or transfer of power from a direct current supply to alternating current form. This type of converter is usually called DC – AC converter or inverter.

(D) **AC-AC Converters** - Circuits which change a fixed a.c. voltage into a variable a.c. supply, i.e., transfer of power from an alternating current supply directly into an alternating current load of different voltage level at fixed frequency or variable frequency. These converters are usually of two types:

(i) AC choppers: change the a.c. supply voltage magnitude only keeping the frequency unchanged.

(ii) Cycloconverters or matrix converters: change the a.c. supply directly to a variable a.c. supply both in magnitude and frequency (also called frequency changers).

1.4 Major Power Electronics Applications

The development of power semiconductors with very high voltage and current ratings has enabled the use of power electronic converters for many applications, such as,

- ***Residential***

Lighting, Space heating, Air conditioning, Cooking, Refrigeration, Power supplies for consumer products and Computers.

- ***Transportation***

Electric trains, Trams, Hybrid electric and all-electric vehicles also utilize controlled power electronic converters for interfacing the battery and motor/generator, Battery chargers, Subways, Conveyor systems, and Rapid transit systems.

- ***Commercial***

Heating, ventilating, and air-conditioning (HVAC) , Refrigeration , Lighting Computers & office equipment, Uninterruptible power supplies (UPS) and Elevators.

- ***Utility Systems***

High-voltage DC transmission (HVDC) , Static VAR compensation, Renewable energy sources (wind, PV, fuel cells), Boiler feed systems , and Energy storage.

- ***Industrial***

Lighting, Pumps, Compressors, AC & DC motor drives, paper industries , steel industries, Gas analysers , Electro plating , Induction heating ,Machine tools, Arc and induction furnaces, Welding, Blowers and fans, Robotic applications, and Process plants.

- ***Aerospace***

Satellite power systems, Aircraft power systems, and Aircraft controls.

- ***Communications and Telecommunications***

Battery chargers, UPS systems, Satellite antenna drive systems in ground station, High power transmitters and receivers, and Power supplies.

Lecture 2.
Power semiconductor devices (Power switches)

Power semiconductor switches are the work-horses of power electronics (PE). There are several power semiconductors devices currently involved in several industrial applications. PE switches works in two states only: Fully on (conducting,
and Fully off (blocking) .
Switches are very important and crucial components in power electronic systems

- What is a good power switch:
  » No power loss when ON
  » No power loss when OFF
  » No power loss during turning ON or OFF
  » Little power required to turn it ON or OFF
  » Bi- directional?
  » Adequate voltage and current ratings
  » Low Turn-on and Turn-off times

- Basic ratings:
  » carrying current, blocking voltage.
  » Speed. Any real device requires a definite time to switch.
  » Second-order ratings: di/dt, dv/dt, momentary capabilities.
  » Power loss
  » Thermal ratings –from power switching devices to heat sink
  » Control ratings: how to operate the switch

TYPES OF POWER SEMICONDUCTOR SWITCHES

The main types of power semiconductor switches in common use are

1. Power Diodes

2. Thyristor devices

   a. Silicon controlled rectifier (SCR)
   b. Static induction thyristor (SITH)
   c. Triac (Triode ac switch)

d. Gate turn-off thyristor (GTO)
e. Mos- controlled thyristor (MCT)
f. integrated gated-commutated thyristor (IGCTs)

3. Power transistors
a. Bipolar junction transistor (BJT)
b. Metal oxide semiconductor field effect transistor (MOSFET)
c. Insulated gate bipolar transistor (IGBT)
d. Static induction transistor (SIT)

2.1 Power Diodes:

These are two terminal switches, as shown in Fig. 2.1 -a, formed of a pn junction. It is not controllable and its operating states are determined by the circuit operating point. When diode is forward biased, it conducts current, i.e a forward positive voltage Vo will turn it on. When it reversed biased (a reverse negative current from Cathode to Anode) will turn it off. With forward biasing a small forward voltage (V_F) will appear across it (0.2-0.3V). Practically, the diode characteristic consists of two regions, as shown in Fig. 2.1 -b; a forward bias region (ON state) where both v_D and i_D are positive and the current in this region increases exponentially with the increase in the voltage, and a reversed bias region (OFF state) where both v_D and i_D, are negative and very small leakage current (μA to mA) flows through the diode until the applied reverse voltage reaches the diode's breakdown voltage limit VBR. Ideally, the diode is represented by a short circuit when forward biased and as an open circuit when reversed biased with the ideal characteristic shown in Fig. 2.1-c.

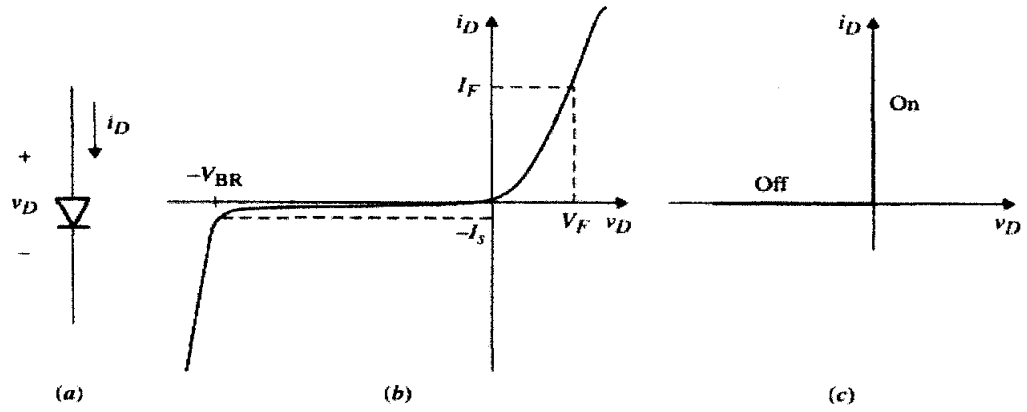


Fig.2.1 Diode: a) symbol, b) characteristic, and c) ideal characteristic .

Types of Power Diodes:

1. Line frequency (general purpose) :
   - on state voltage very low (below 1 V)
   - large reverse recovery time t_{rr} (about 25μs) .
   - very high current (up to 6 kA) and voltage (8 kV) ratings
   - used in line-frequency (50/60Hz) applications such as rectifiers.
   - Slow recovery (Fig.2.2)

t_{rr} =reverse recovery time

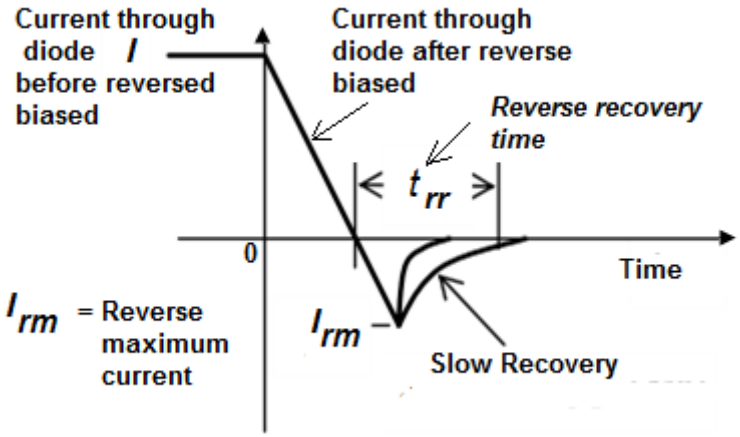


Fig.2.2

2. Fast recovery:
   - very low t_{rr} (<1 μs).
   - power levels at several hundred volts and several hundred amps.
   - normally used in high frequency circuits.

3. Schottky :
   - very low forward voltage drop (typical 0.3V)
   - limited blocking voltage (50-100V) .
   - used in low voltage, high current application such as switched mode power supplies.

2.2 Thyristors (Silicon Controlled Rectifiers "SCRs")

The thyristor ,it is also called silicon controlled rectifier (SCR), is a four-layer, three terminal switching semiconductor device, with each layer consisting of an alternately N or P-type material, for example N-P-N-P. The main terminals, labeled anode and cathode, are across the full four layers, and the control

terminal, called the gate, is attached to one of the middle layers as shown in Fig.2.3.

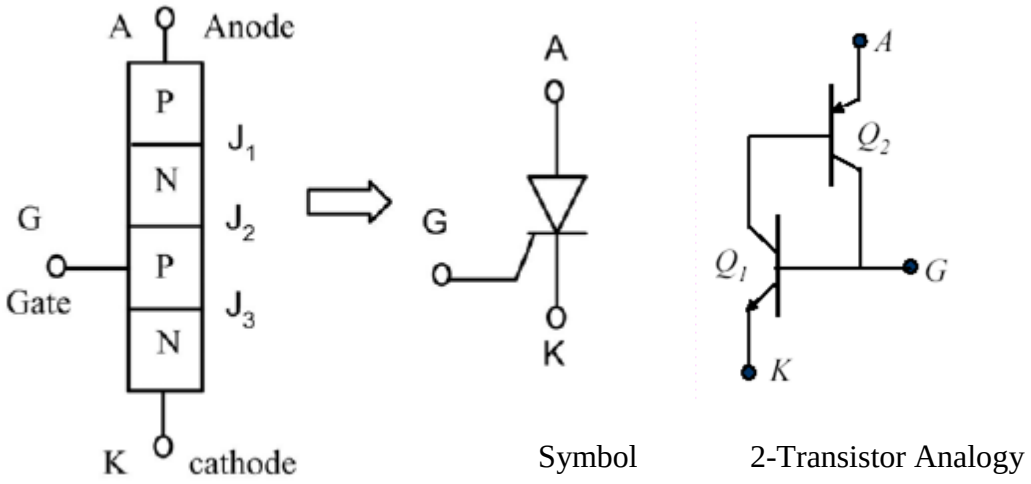


Fig.2.3: The thyristor and its equivalent 2-transistor circuit.

SCRs are mainly used where high currents and voltages are involved, and are often used to control alternating currents, where the change of sign of the current causes the device to automatically switch off. Like a diode, an SCR conducts only in one direction.The SCR symbol and its ideal characteristic are shown in Fig,2.4

Modern SCRs can switch large amounts of power (up to megawatts). In the realm of very high power applications, they are still the primary choice.

SCR is a controllable switch that usually required to be latched to conduct. This latching (triggering) process is carried out by injecting current to the gate terminal (i_g),Fig.2.5, at the required latching instant provided that the device is forward biased (v_{AK} is positive). Practically, the thyristor characteristic has three main regions as shown in Fig. 2.6;

- the Conduction Region where the thyristor is operating in its ON state,
- the Forward Blocking Region where the thyristor is forward biased but not yet triggered or the voltage didn't reach the forward breakover voltage, and
- the Reverse Region that consists of the reverse blocking region and the reverse avalanche region similar to the diode characteristic.

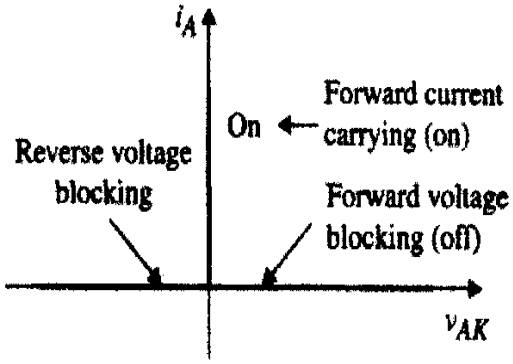
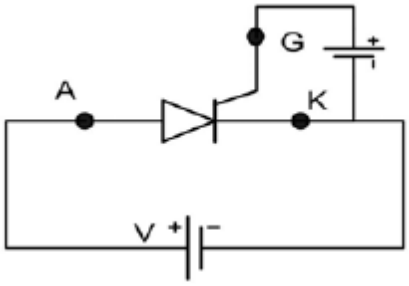


Fig. 2.4 Thyristor:(a) symbol, (b) ideal characteristic ,

Fig.2.5 Thyristor gate circuit



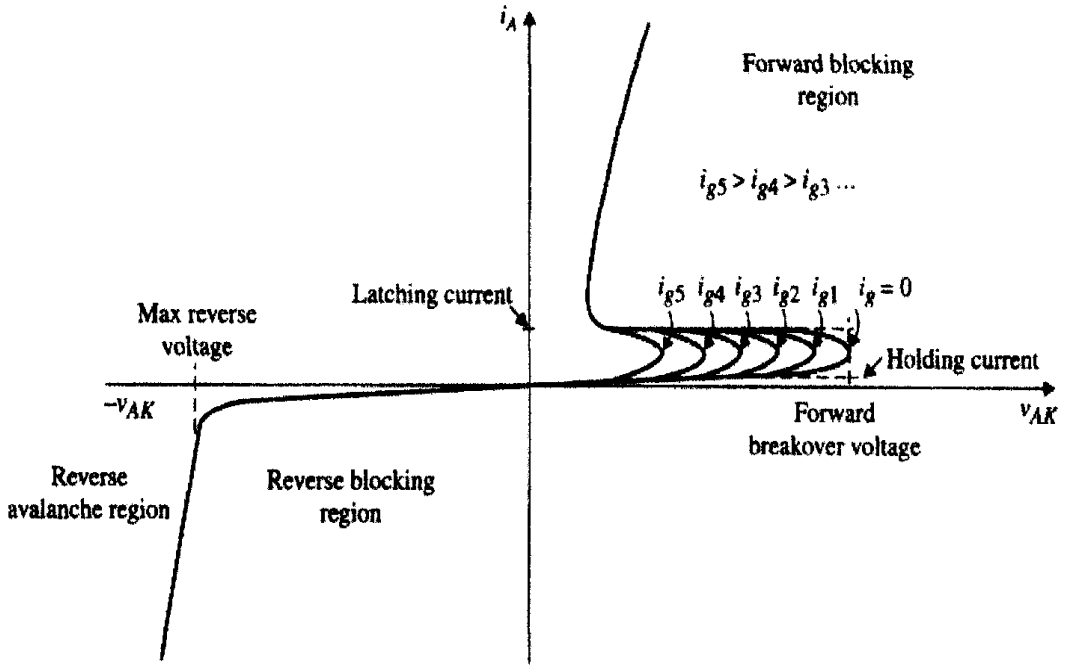


Fig. 2.6 Thyristor (SCR) characteristic

Thyristors can only be turned on with three conditions:

1. The device must be forward biased, i.e., the anode should be more positive than the cathode.
2. A positive gate current *(Ig*) should be applied at the gate.
3. the current through the thyristor should be more than the latching current.

Once conducting ,the anode current is LATCHED (continuously flowing).

Important points for the SCR characteristic:

o **Latching Current:** This is the minimum current required to turn on the SCR device and convert it from the Forward Blocking State to the ON State.
o **Holding Current:** This is the minimum forward current flowing through the thyristor in the absence of the gate triggering pulse.
o **Forward Breakover Voltage:** This is the forward voltage required to be applied across the thyristor to turn it ON without the gate signal application.
o **Max Reverse Voltage:** This is the maximum reverse voltage to be applied across the thyristor before the reverse avalanche occurs.
Ideally, SCRs are represented by a short circuit when operating within the conduction region and as an open circuit when operating within the blocking region. The ideal characteristic is shown in Fig. 2-b. It is also worth mentioning that once the SCR is triggered and turned ON the gate signal can be removed without turning it OFF. SCRs are turned OFF when reversing the terminal voltage and current.

Turning on/off mechanism

- Thyristor (SCR) Conduction

- In reverse -biased mode, the SCR behaves like a diode. It conducts a small leakage current which is almost dependent of the voltage, but increases with temperature. When the peak reverse voltage is exceeded, avalanche breakdown occurs, and the large current will flow.

- In the forward biased mode, with no gate current present (i.e. in the untriggered state, the device exhibits a leakage current. If the forward breakover voltage (Vbo) is exceeded, the SCR “self-triggers” into the conducting state and the voltage collapses to the normal forward volt-drop, typically 1.5-3V. The presence of any gate current i_g will reduce the forward breakover voltage and will trigger the thyristor at any required instant (α), see Fig.2.7.

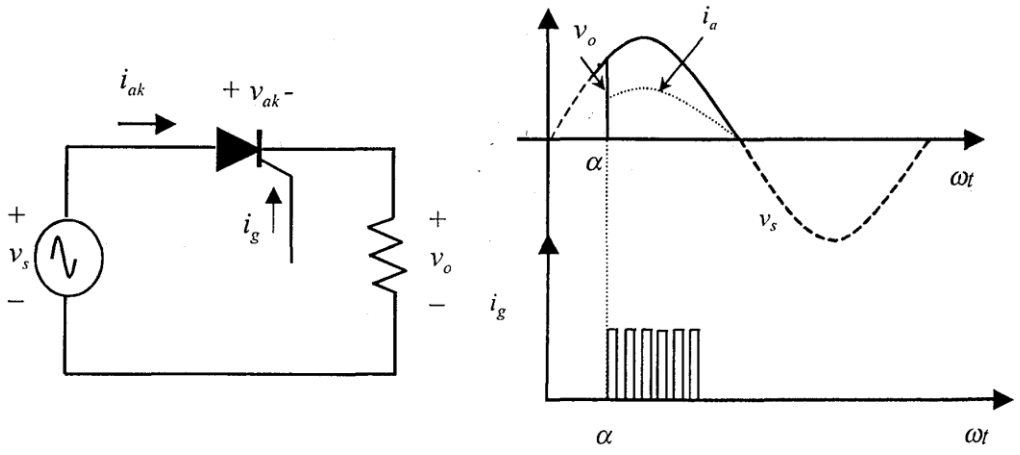


Fig.2.7

- Thyristor turn off

The process of turning OFF SCR is defined as "Commutation". Thyristor cannot be turned off by applying negative gate current. It can only be turned off if the current *I* through it goes negative (reverse). In all commutation techniques, a reverse voltage is applied across the thyristor during the turn OFF process.
There are two methods by which a thyristor can be turned OFF.

i. Natural Commutation
ii. Forced Commutation

Natural Commutation

In AC circuit, the current always passes through zero for every half cycle. As the current passes through natural zero, a reverse voltage will simultaneously appear across the device. This will turn OFF the device immediately. This happens when negative portion of the of sine-wave occurs. This process is called as "**natural commutation**" since no external circuit is required for this purpose.

Forced Commutation

Another method of turning off is known as "**forced commutation**". The anode current is “diverted” to another circuitry. To turn OFF a thyristor, the forward anode current should be brought to zero for sufficient time to allow the removal of charged carriers. In case of DC circuits the forward current should be forced to zero by means of some external circuits.

Forced commutation circuit

Fig.2.8 shows one typical thyristor commutation circuit. Thyristor T_m is the main Thyristor through which the flow of power is controlled .Capacitor C and the four Thyristors (T_1,T_2 ,T_3 ,T_4) is the commutation circuit . The function of the commutation circuit is to switch off the main thyristor at the end of each ON period. During ON period of the thyristor, the two auxiliary thyristors T_2 and T_4 are triggered so that the capacitor C is charged such as plate a is positive. To switch OFF, T_m thyristors (T_1,T_3) are triggered ON. This results in applying reverse polarity voltage across T_m and hence it will be switched OFF. Also the capacitor polarity will be reversed; i.e. plate b will now be positive.
Thyristor T_m is switched ON for the next ON period, and now to switch OFF T_m, Thyristors (T_2,T_4) are switched ON , and so the cycle is repeated .

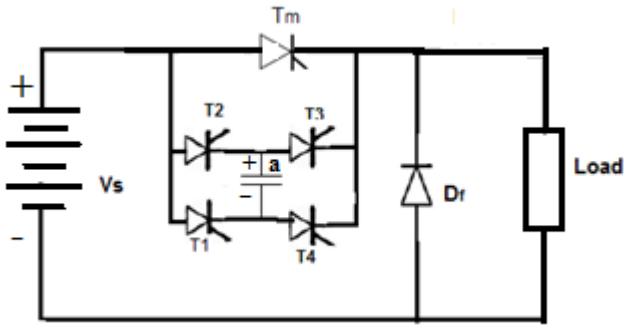


Fig.2.8 Typical forced commutation circuit for a thyristor.

Types of Thyristors

1. **Phase controlled**

- rectifying line frequency voltage and current for ac and dc motor drives .
- large voltage (up to 7 kV) and current (up to 5 kA) type1 capability .
- low on-state voltage drop (1.5 to 3V).

2. **Inverter grade**

- used in inverter and chopper
- Quite fast. Can be turned-on using “force commutation” method.

3. **Light activated**

- Similar to phase controlled, but triggered by pulse of light .
- Normally very high power ratings.

SCR ratings for voltage and current approach those of diodes. Devices for high-voltage dc (HVDC) conversion have been built with simultaneous 12 kV and 6 kA type 2 ratings. Figure 2.9 below shows types of thyristors in practice.

Fig . 2.9 Power semiconductor switches (thyristors).

2.3 The Triac

The triac is a two thyristors connected back – to – back, used for high or medium power control for both a.c and d.c applications, as shown in Fig.2.10. Either of the electrodes MT1 and MT2 can act as anode and either is cathode. The device can be triggered by either positive or negative voltage on the gate with respect to MT1.This device is effectively two thyristors (SCR s) back- to-back in construction with an external n-region which is the gate.

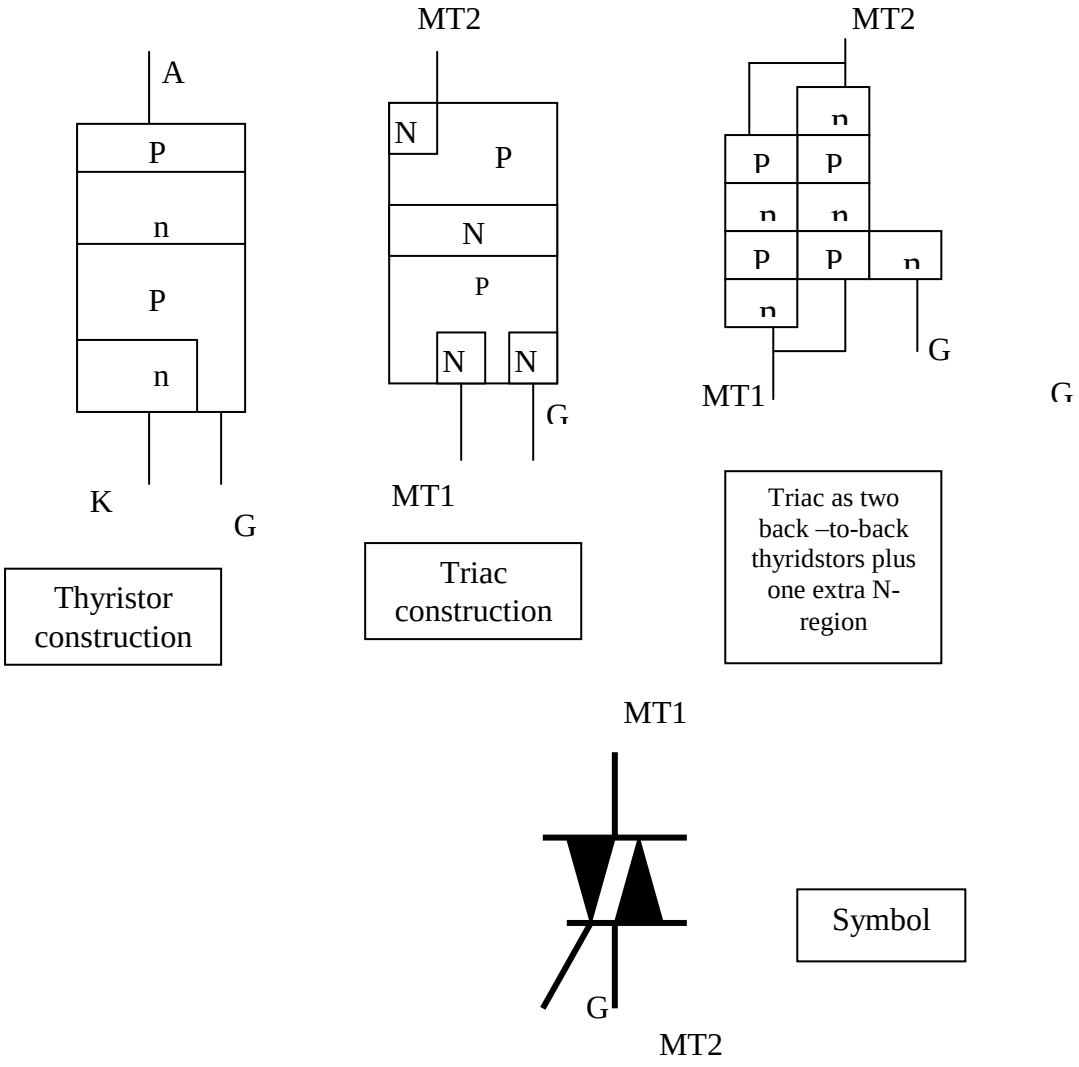


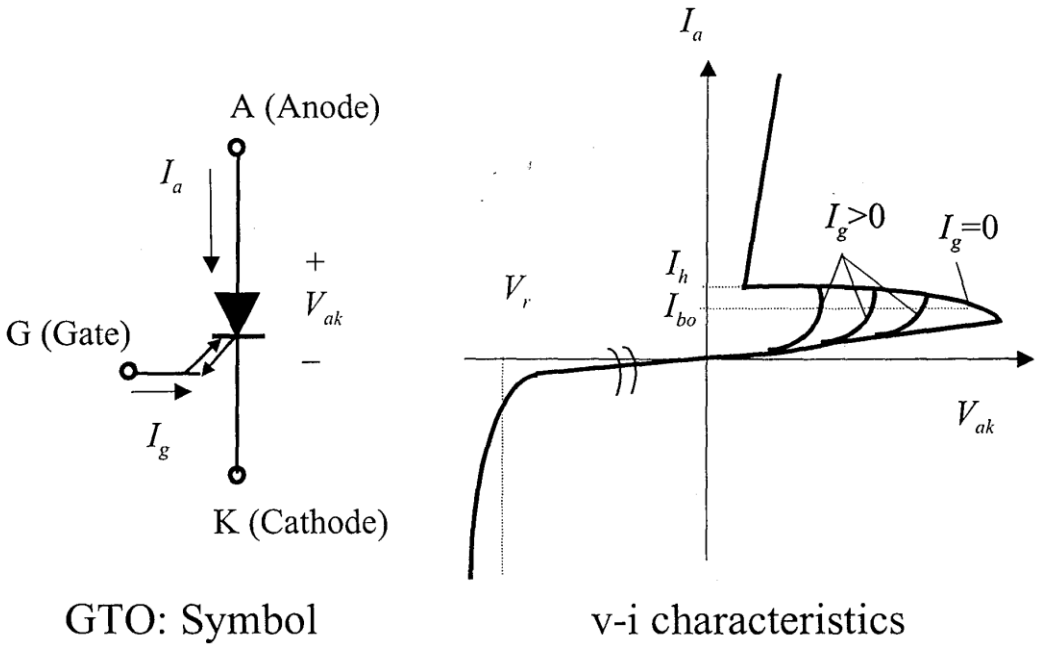
Fig.2.10

The four possible mode of operation of triac are

(a) MT2 +, G + (both relative to MT1) Gate current flows into gate terminal.
(b) MT2+, G – Gate current opposite to (a).
(c) MT2- , G + Gate current as (a)
(d) MT2- , G – Gate current as (b)

2.4 Gate Turn Off Thyristors (GTO Thyristors),

GTO behaves like normal thyristor, but can be turned off using gate signal. However turning off is difficult. Need very large reverse gate current (normally 1/5 of anode current).GTO symbol and characteristics are shown in Fig.2.11.



GTO: Symbol v-i characteristics

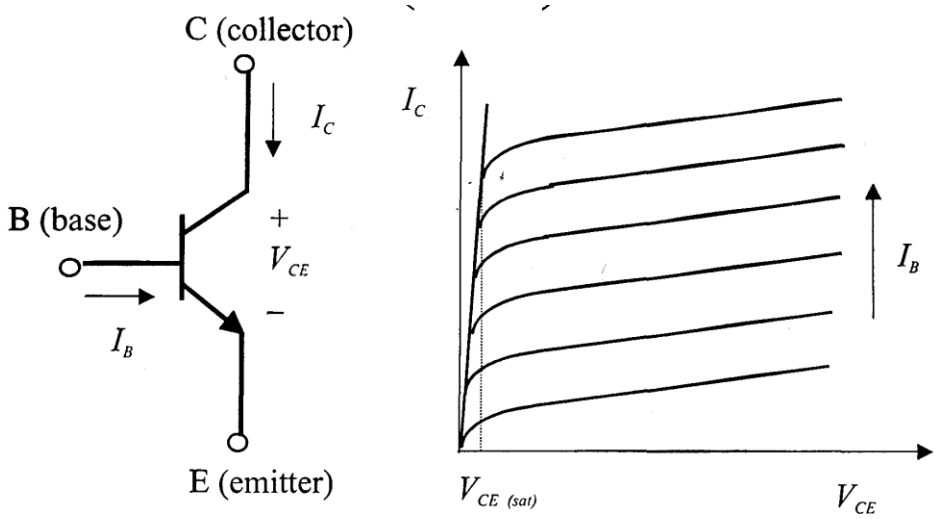
Fig.2.11 Gate turn off thyristor

- Ratings: Voltage: V_{ak} < 6kV, Current: I_a < 2 kA -
- Frequency < 5 kHz.
- Gate drive design is very difficult. Need very large reverse gate current to turn off.
- GTO normally requires snubbers. High power snubbers are expensive.
- In very high power region (> 6kV, > 2kA), development in gate-controlled thyristor (GCT) may effectively end the future of GTO.

2.5 Power transistors

Can be turned "ON"and "OFF" by relatively very small control signals.
Operated in SATURATION and CUT OFF modes only. No "linear region" operation is allowed due to excessive power loss.
Traditional devices: Bipolar junction transistors (BJT), Metal oxide silicon field effect transistor (MOSFET), Insulated gate bipolar transistors (IGBT).
Emerging (new) devices: Gate controlled thyristors (IGCT).

1- Bipolar Junction Transistor (BJT)



BJT: symbol (npn) v-i characteristics

Fig.2.12 BJT power transistor

- Ratings: Voltage: V_{CE} < 1600 V, Current I_C < 1000 A.
- Switching frequency up to 5 kHz
- Low on-state voltage V_{CE}: 2-3V, hence low on – state loss.
- Low current gain (β) Need high base current to obtain reasonable I_C.
- Expensive and complex base drive circuit.
- Not popular in new products.

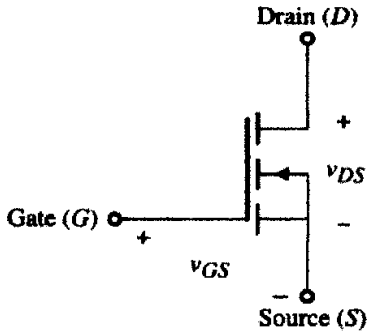
BJT Power Transistor Characteristics

-To turn on/off the device, a base drive circuit is connected to the base and emitter terminal.
-To turn on, current is injected into the base terminal. When turned on, conventional current passes from collector to emitter.
-To turn-off, the base current is removed.
-The current gain of a BJT ends to be low when operated in the saturated ON condition. β< 10 is common. It deteriorates as voltage ratings increases.
-It is normal to use Darlington connection for higher current gain.

2- Metal Oxide Semiconductor Field Effect Transistors "MOSFETs"

These are three terminal switches as shown in Fig. 2.13.

Fig.2.13 MOSFET:symbol (n-channel)



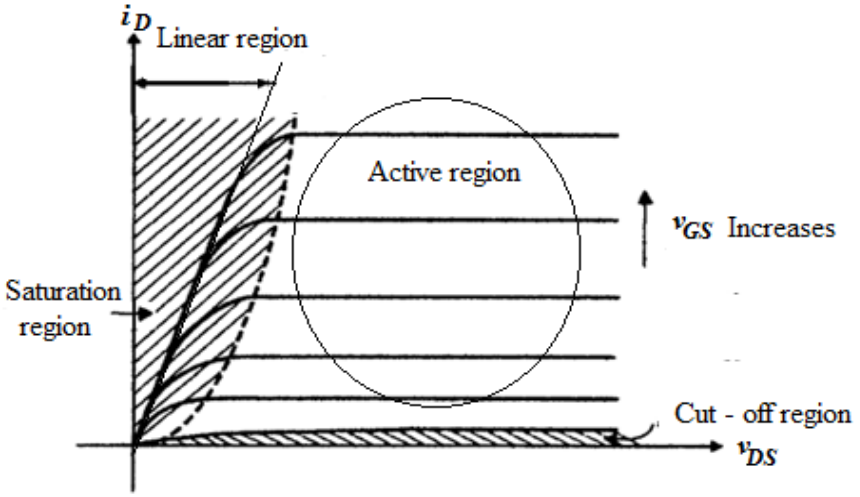
This is considered the fastest power switching device (200 kHz) for rating voltages < 500V, current I_{DS} < 300A ,or at 100 kHz , < 1500 V , 300 A .
MOSFET characteristics

- Turning on and off is very simple. Only need to provide V_{GS} = +15V to turn on and 0V to turn off. Gate drive circuit is simple.
- Basically low voltage device. High voltage device are available up to 600V but with limited current. Can be paralleled quite easily for higher current capability.
- Dominant in high frequency application (>100 kHz). Biggest application is in switched-mode power supplies.
- On state loss relatively high. V_{DS} > 3 V.

Practically, MOSFET's characteristic consists of three regions, as shown in Fig.2.13 :
- Cut OFF region (OFF state) when $V_{GS} < V_{Th.}$
- Linear region when $V_{DS} < V_{GS} - V_{Th}$, and
- Active region when $V_{DS} > V_{GS} - V_{Th}$.

Ideally, MOSFETs are represented by a short circuit when operating within the ON State and as an open circuit when operating within the OFF State.

Fig.2.13 MOSFET characteristics



3 - Insulated Gate Bipolar Transistor "IGBT"

This is also a three terminal switch as shown in Fig. 2.14. Its operation modes and characteristics are almost similar to those for MOSFETs, shown in Fig. 2.13, except for the operating ranges.

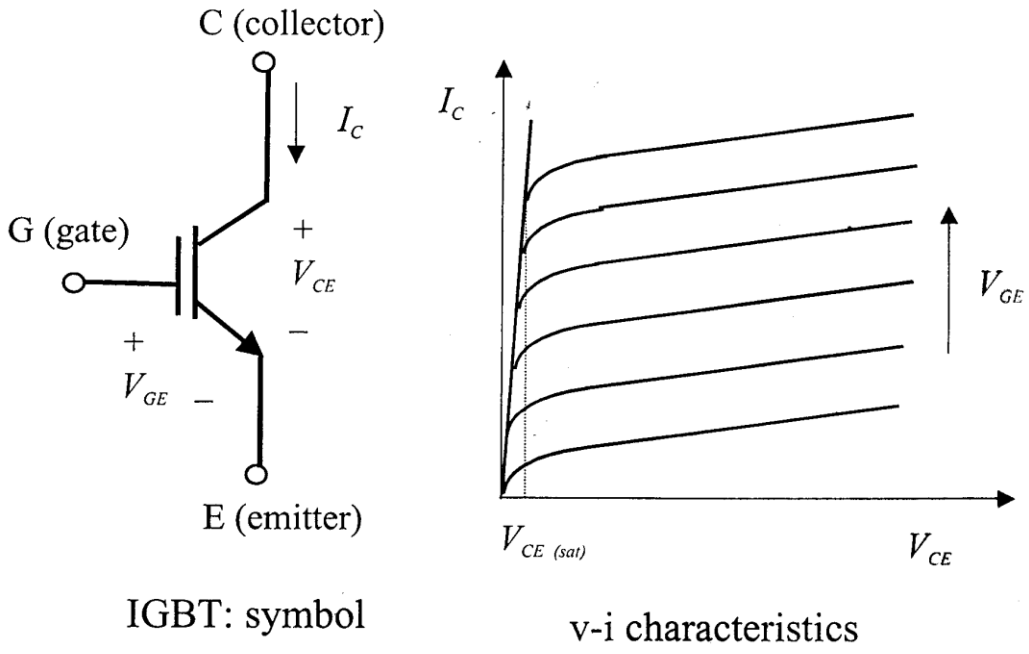


Fig.2.14: IGBT

IGBT has a combination of BJT and MOSFET characteristics. Compromises include:

- Gate behavior similar to MOSFET - easy to turn on and off.
- Low losses like BJT due to low on-state Collector-Emitter voltage V_{CE} = (2-3V).
- Ratings: Voltage: V_{CE} < 6000V, Current 2500A currently available.

- Good switching capability (up to 100 kHz) for newer devices.

- Typical application, IGBT is used at 20-50 kHz.
- For very high power devices and applications, frequency is limited to several kHz -
- Very popular in new products; practically replacing BJT in most new applications
- .Snubberless" operation is possible. Most new IGBTs do not require snubber.

Other switching devices:

There are several other power switching device available such as: Diac , Static Induction Transistors (SITs), Static Induction Thyristors (SITHs), and MOS-Controlled Thyristors (MCT).

MCTs (MOS Controlled Thyristor)

An effort to combine the advantages of bipolar junction and field-effect structures has resulted in hybrid devices:

- The IGBT is an improvement over a BJT.
- The MCT is an improvement over a thyristor.

MCT characteristics

- MCT can be switched on or off by negative or positive gate voltage, respectively.
- It has fast TURN-ON and then OFF times, with high-speed switching capability.
- Low conduction losses, low switching losses, and high current density.
- The gating requirements of an MCT are easier than those of the GTO since it needs smaller gate turn-off current due to its high gate input impedance.
- Compared with the power MOSFET, it has higher current density and lower forward drop . It has great potential in high-power, high-voltage applications. A peak power of 1 MW can be switched off in 2 *ns* by a single MCT.

Therefore, the MCT overcomes several of the limitations of the existing power devices and promises to be a better switch for the future. The MCT symbol and equivalent circuit are shown in Fig.2.15. The MCT characteristics are shown in Fig.2.16.

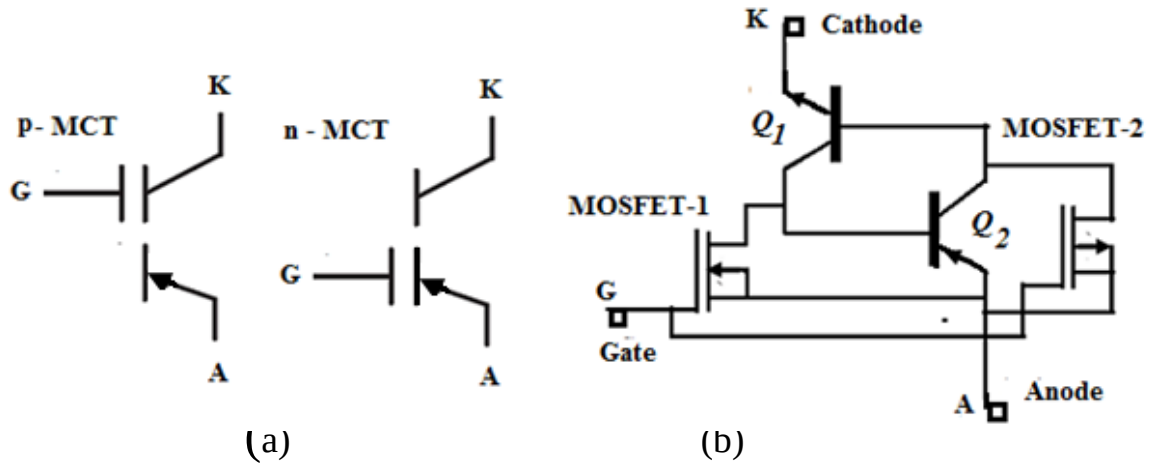
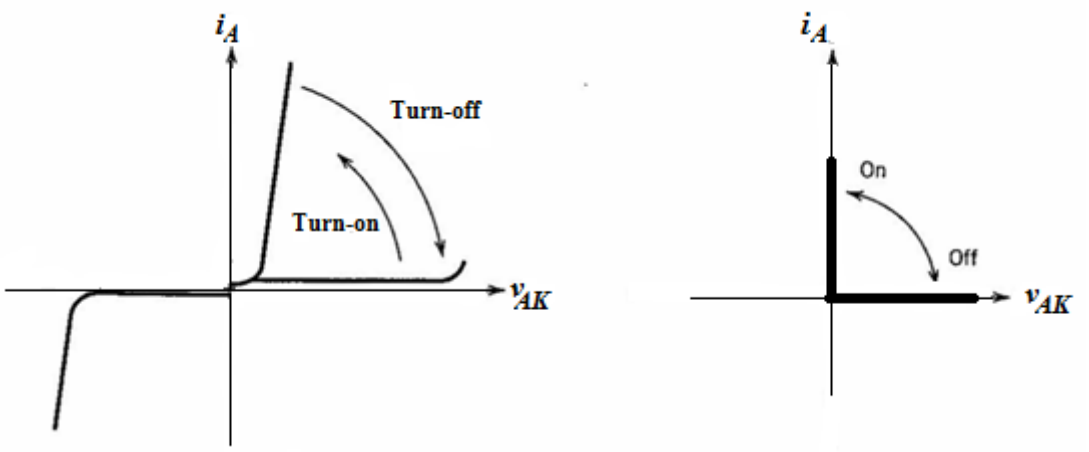


Fig. 2.15 MCT (a) symbol, (b) equivalent circuit



2.16 MCT (b) *v-i* characteristic, (c) idealized characteristic.

New Emerging Devices

The Power Silicon Carbide Semiconductors

Silicon Carbide (SiC) semiconductors are a new option for power electronic designers looking to improve system efficiency, its features are:

Extremely low Switching losses

• Zero reverse recovery charge - improves system efficiency

High Power Density

• Smaller footprint device reduces system size and weight

High thermal conductivity
• 2.5 *x* more thermally conductive than silicon
Reduced Sink requirements
• results in lower cost and smaller size
High temperature operation
• increased power density and improved reliability.

Figure 2.17 shows a summary of power semiconductor device capabilities by the year 2016.

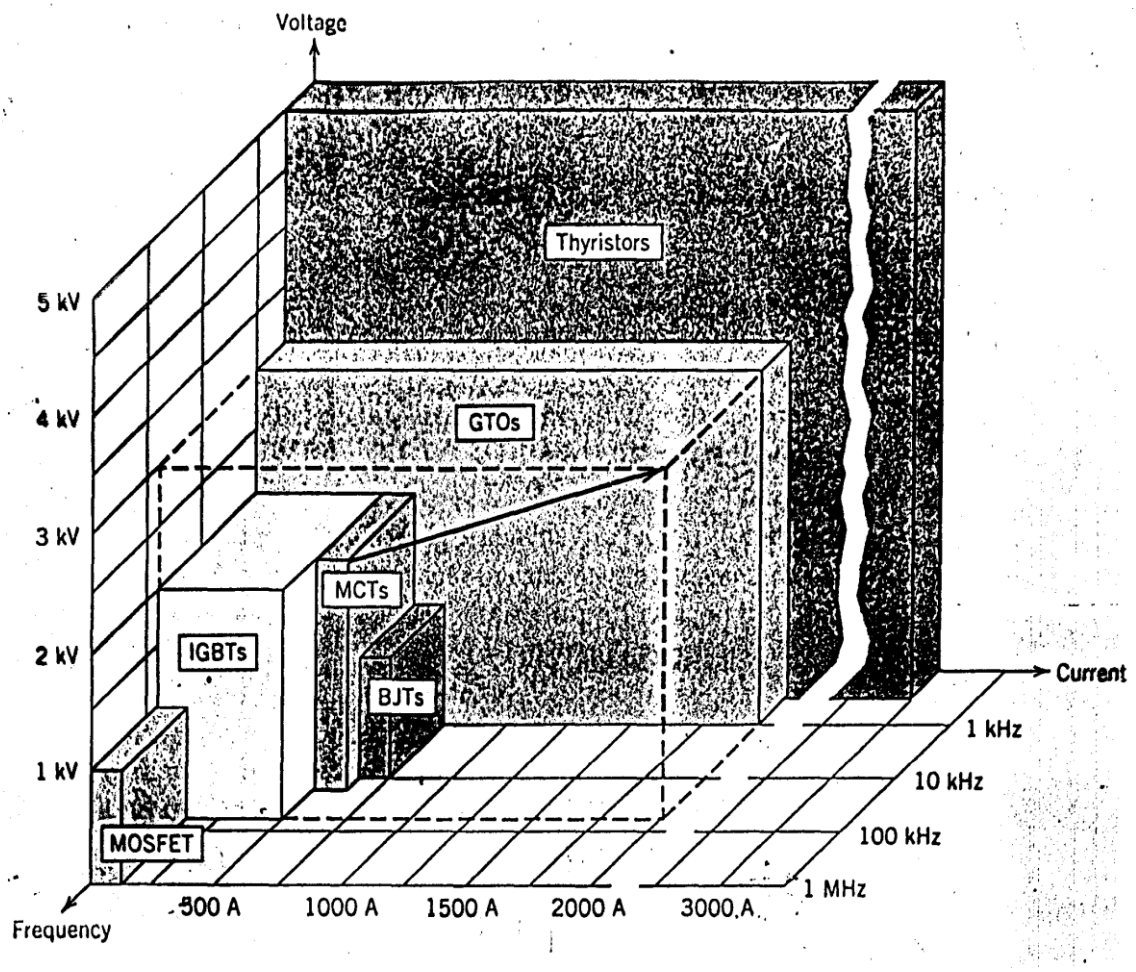


Figure 2.17 : Summary of power semiconductor device capabilities. All devices except the MCT have a relatively mature technology, and only evolutionary improvements in the device capabilities are anticipated in the next few years. However, MCT technology is in a state of rapid expansion, and significant improvements in the device capabilities are possible as indicated by the expansion arrow in the diagram.

LECTURE .3 : AC-DC CONVERSION

(RECTIFICATIONS)

3.1Basic Rectifier Circuits

Several types of rectifier circuits are available: single-phase and three-phase half-wave and full-wave, controlled and uncontrolled, etc. For a given application, the type used is determined by the requirements of that application. In general the types of rectifiers are:

1. Uncontrolled Rectifier
   Provide a fixed d.c. output voltage for a given a.c. supply where diodes are used only.

2. Controlled Rectifier
   Provide an adjustable d.c. output voltage by controlling the phase at which the devices are turned on, where thyristors and diodes are used.

   A. *Half-controlled*
   allows electrical power flow from a.c. to d.c. (i.e. rectification only)

   B. *Fully - controlled*
   allow power flow in both directions (i.e. rectification and inversion)

Application of Rectifier Circuits

- There are many applications for rectifiers. Some of them are:
  »Variable speed dc drives,
  »Battery chargers,
  »DC power supplies for electric railways and
  »Power supply for a specific application like electroplating

- Power rating of a single-phase rectifier tends to be lower than 10 kW.

- Three-phase bridge rectifiers are used for delivering higher power output, up to 500 kW at 500 V dc or even more.

In our study, we shall be interested mainly in single-phase half-wave and full—wave phase controlled rectifiers, but we shall a1so consider uncontrolled diode rectifiers to clarify the presentation.

NOTE 1: In the following analysis we shall neglect forward vo1tge drop, and reverse and forward leakage currents in diodes and thyristors, we shall also neglect transient switching effects related to the dynamic characteristics of the solid-state components

NOTE 2: We shall always assume that the pulse is of sufficient amplitude to switch on the appropriate thyristor.

3.2 Uncontrolled Rectifications

In this type, the generated DC power is fixed with the converter used and the input AC power. They usually use diodes as their power switches. The following subsections deal with the basic operation of some examples of uncontrolled rectifiers single-phase half-wave rectifier loaded with resistive and resistive inductive loads.

3.2.1- Single-phase Half-Wave Uncontrolled Rectifier with resistive load

Fig.3.1 shows the basic circuit for a single-phase, half-wave uncontrolled rectifier supplying a resistive load. The circuit is supplied by a single phase transformer whose secondary represents the rectifier's circuit AC source (v_s) that is represented by a sinusoidal wave given by

$$v_s = V_m \sin(\omega t) \qquad (3.1)$$

where v_S is the supply voltage, V_m is the peak value of the supply voltage, ω is the angular frequency, and t and is the time.

For this configuration, the diode will conducts (becomes forward biased) whenever the supply voltage (v_S) is positive.

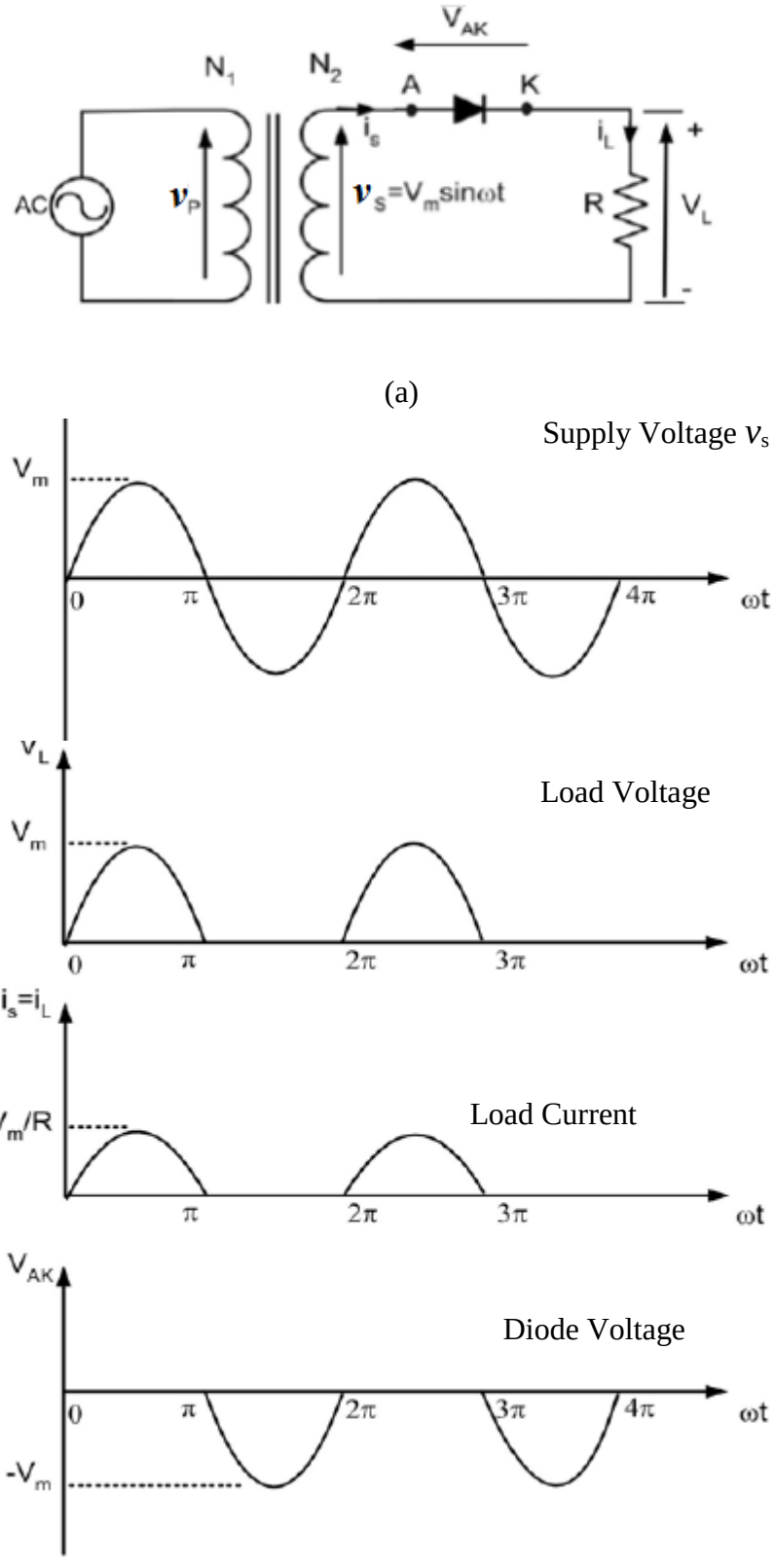


Fig.3. 1 Single-phase half-wave rectifier: a) circuit and b) waveforms

The average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{2\pi}\int_0^{\pi} v_s\,(\omega t)d\omega t \tag{3.2}$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m sin\omega t\, d\omega t = \frac{V_m}{2\pi}\,(-cos\pi + cos0^o) = \frac{V_m}{\pi} \tag{3.3}$$

Since the load is resistive, therefore the load voltage and current are in phase and they are related by $i = v / R$. Consequently, the average value of the load current I_{dc} is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R} \tag{3.4}$$

The output DC power is given by:

$$P_{dc} = V_{dc}I_{dc} = \frac{V_{dc}^2}{R} \tag{3.5}$$

The rms value of the load voltage V_{orms} can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} v_s^2\,(\omega t)d\omega t} \tag{3.6}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_m sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{2\pi}\int_0^{\pi} \frac{1}{2}(1 - cos2\omega t)d\omega t} = \frac{V_m}{2} \tag{3.7}$$

Since: $(sin\omega t)^2 = \frac{1}{2}(1 - cos2\omega t)$

Therefore the rms value of the load current I_{orms} is :

$$I_{orms} = \frac{V_{rms}}{R} = \frac{V_m}{2R} = \frac{I_m}{2} \tag{3.8}$$

The rms value $I_{orms} = I_m/2$ for half – wave operation compared with the corresponding value for pure sinusoidal operation which is $I_{orms} = I_m/\sqrt{2}$.

The output a.c. power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.5V_m)^2}{R} \quad (3.9)$$

Performance parameters

The performance of a rectifier can be evaluated in terms of the following parameters:

The output d.c. power is given by

$$P_{dc} = V_{dc} I_{dc}$$

The output a.c. power is given by

$$P_{ac} = V_{orms} I_{orms}$$

3. The efficiency of the converter is given by

$$\eta = \frac{P_{dc}}{P_{ac}} \quad (3.10)$$

4. Output A.C. Component

The output voltage can be considered to have two components: including (i) d.c. value and (ii) the a.c. components or ripple. The rms value of the a.c. component of the output voltage is

$$V_{ac} = \sqrt{V_{orms}^2 - V_{dc}^2} \quad (3.11)$$

5. Form Factor, *FF*

It is a measure of the shape of the output voltage.

$$FF = \frac{V_{orms}}{V_{ac}} \quad (3.12)$$

6. Ripple factor, *RF*

It is a measure of the ripple content or the degree of distortion in a rectified voltage waveform which can be calculated as

$$\text{Ripple factor} = \frac{\text{rms value of the ac components}}{\text{average value}}$$

$$= \frac{V_{ac}}{V_{dc}} = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} \quad (3.13)$$

7. Transformer utilization factor, *TUF*

$$TUF = \frac{P_{dc}}{V_s I_s} \qquad (3.14)$$

where

Vs : rms voltage of the transformer secondary
Is : rms current of the transformer secondary

Example 3.1

An ideal single-phase source, 240 V, 50 Hz, supplies power to a load resistor $R = 100\ \Omega$ via a single ideal diode.
(a) Calculate the average and rms values of the current and the power dissipation.
(b) Calculate the circuit power factor and the ripple factor.
(c) What must be the rating of the diode?

Solution:

$$I_{dc} = \frac{V_m}{\pi R} = \frac{240\sqrt{2}}{\pi \times 100} = 1.08A$$

$$I_{orms} = \frac{V_m}{2R} = \frac{240\sqrt{2}}{2 \times 100} = 1.7A$$

The average power dissipation in the load resistor *R* is given by the average a.c. power:

$$P_{ac} = I_{orms}^2 R = \left(\frac{V_m}{2R}\right)^2 = \frac{{V_m}^2}{4R^2} = (1.7)^2 \times 100 = 289\ \text{W}$$

$$P_{in} = V_s \times I_{orms} = 240 \times 1.7 = 408\ \text{W}$$

$$PF = \frac{P_{ac}}{P_{in}} = \frac{289}{408} = 0.708$$

$$\text{Ripple factor} = \text{RF} = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = \frac{\sqrt{I_{orms}^2 - I_{dc}^2}}{I_{dc}}$$

$$\text{RF} = \sqrt{(\frac{I_{orms}}{I_{dc}})^2 - 1} = \sqrt{(\frac{1.7}{1.08})^2 - 1} = 1.21$$

The value 1.25 for half – wave rectification is large, since the ideal value of the ripple factor should be zero for the output d.c. voltage. The diode must be rated in terms of a peak reverse voltage and a mean forward current.

Diode PRV $= V_m = \sqrt{2} \times 240 = 339.4V$, Choose 400V diode

Either the rms or the mean (average) current could be use as the basis of current rating: Since I_{orms} = 1.7A choose 2A diode rating.

3.2.2 . Single –phase half - wave uncontrolled rectifier with (R-L) Load:

If the load consists of a series resistor and inductor, the current will flow through the negative cycle as well; Fig.3.2 shows the circuit diagram and Fig.3.3 shows the load voltage and current waveform for this case.

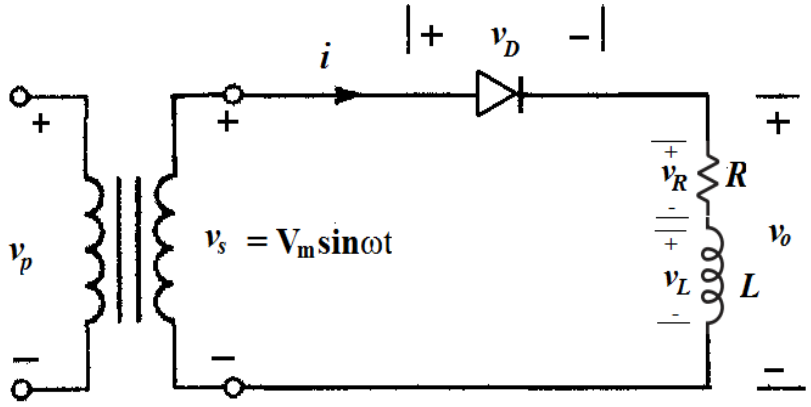


Fig .3.2 Single – phase half – wave rectifier with *R –L* load.

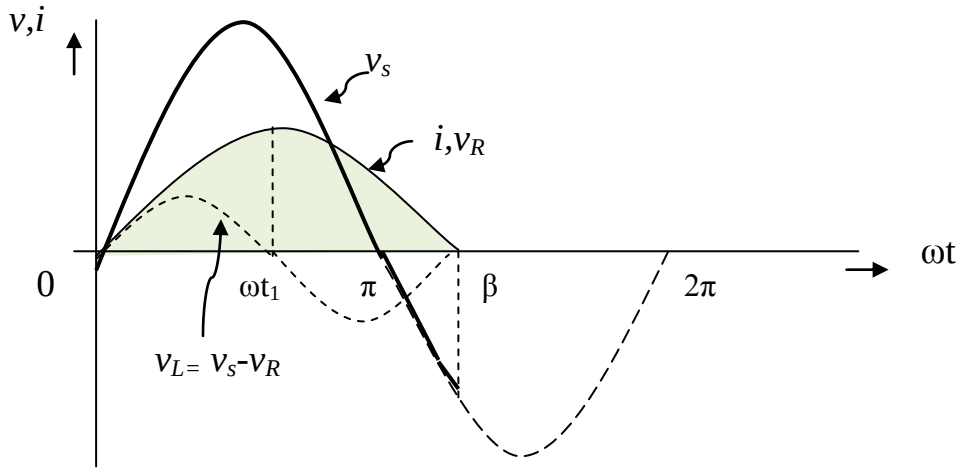


Fig.3.3 Waveforms for Fig.3.2.

During conduction period, we have by KVL;

$$v_s = v_o = v_R + v_L \qquad (3.15)$$

$$V_m sin\omega t = i\,R + L\frac{di}{dt} \qquad (3.16)$$

Each supply period (cycle) in Fig. 3.3 can be divided into 4-distinct regions:-

- From 0- ωt_1: the current rises from zero to peak, which lags the voltage peak due to circuit inductance; v_L is positive and the inductance store energy.
- From ωt_1 -π: the current decays, and hence v_L is negative. Both source and inductance supply energy to *R*.
- From π - β: the current continues to decay until it reaches zero , v_L remains negative ,and hence energy is supplied by inductance to both source and resistor.
- From β - 2 π: at β current reaches zero and the diode cut- out. Current remains zero until the beginning of the next positive half cycle.

The average output voltage is

$$V_{dc} = \frac{1}{2\pi}\int_0^{\beta} v_s\,(\omega t)d(\omega t) = \frac{1}{2\pi}\int_0^{\beta} V_m sin\omega t\, d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi}\left[-\cos\omega t\begin{Bmatrix}\beta\\0\end{Bmatrix} = \frac{V_m}{2\pi}[-\cos\beta + \cos 0] = \frac{V_m}{2\pi}[1-\cos\beta] \qquad (3.17)$$

The average output current is

$$I_{dc} = \frac{V_m}{2\pi R}[1-\cos\beta] \qquad (3.18)$$

Equation of the current:

The equation for the current through R-L load can be found from the solution of the differential equation (3.16) which can be re-written as:

$$L\frac{di}{dt} + i\,R = V_m sin\omega t \qquad (3.19)$$

This is a first order differential equation. The solution of this equation has two parts:

1- Steady state solution: $i_{ss}(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta)$

2- Transient solution : $i_{tr}(\omega t) = Ae^{-\frac{t}{\tau}}$

where $Z = \sqrt{R^2 + \omega^2 L^2}$;

$$\tan\theta = \frac{\omega L}{R}$$

$$\tau = \frac{L}{R} = \text{time constant}$$

A = constant

The complete solution is : $i(\omega t) = i_{ss}(\omega t) + i_{tr}(\omega t)$

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\frac{t}{\tau}} \qquad (3.20)$$

The constant A can be found from initial conditions: when t = 0, $i(\omega t) = 0$,

$$0 = \frac{V_m}{Z}\sin(-\theta) + Ae^{-0}$$

$$\therefore A = \frac{V_m}{Z}\sin(\theta) \qquad (3.21)$$

Substitute eq. (3.21) into eq. (3.20) yields

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + \frac{V_m}{Z}\sin(\theta)\, e^{-\frac{t}{\tau}}$$

The final current equation is

$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\omega t - \theta) + sin\theta e^{-cot\theta\,.\omega t}] \qquad (3.22)$$

The extinction angle can be determined for a given load impedance angle θ from the above current equation (3.22).

To find β ,

From the final condition of the current, when $= \beta$, $i = 0$, hence from above equation :

$$0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$0 = [\sin(\beta - \theta) + sin\theta e^{-cot\theta\,.\beta}]$$

$$\sin(\beta - \theta) = -\, sin\theta e^{-cot\theta\,.\beta} \qquad (3.23)$$

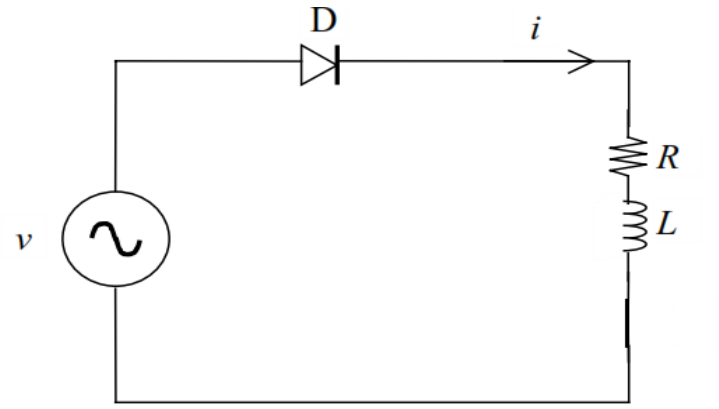
This is a transcendental equation which cannot be solved analytically. It can only be solved numerically by iteration technique as follows:

The initial guess of β is : $\beta_o = 180^{\circ} + \theta + \Delta$, where Δ is few degrees.

Example 3.2

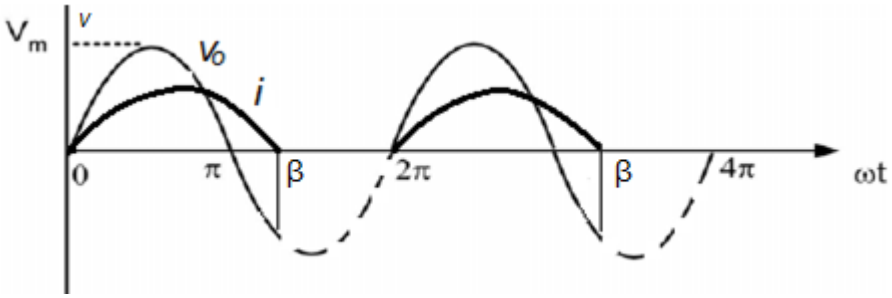
For the half- wave uncontrolled rectifier circuit supplying a series resistive-inductive load shown in Fig.1. The supply voltage is v =300 sinωt and the supply frequency is 50 Hz and the load parameter values are: R =10 Ω, and X_L= 20 Ω.
(a) Sketch the load voltage and current waveforms.
(b) Calculate the current extinction angle β.
(c) Calculate the average DC load voltage V_{dc}.
(d) Calculate the angle of maximum current.

Fig.1



Solution:

(a) The output voltage and current waveforms are:



(b) It is given that:

$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\omega t - \theta) + sin\theta e^{-cot\theta\,.\omega t}]$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ $\tan\theta = \omega L/R$

To find β ,

From the final condition of the current, when $= \beta$, $i = 0$, hence from above equation :

$$0 = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\beta - \theta) + sin\theta e^{-cot\theta \,.\beta}]$$

$$0 = [\sin(\beta - \theta) + sin\theta e^{-cot\theta \,.\beta}]$$

$$0 = \sin(\beta - \theta) + \ sin\theta e^{-cot\theta \,.\beta}$$

This is a transcendental equation which can be solved numerically by iteration technique as follows:

The initial guess of β is : $\beta_o = 180^o + \theta + \Delta$, where Δ is few degrees.

$tan\ \theta = \omega L/R \ = X_L/R = 20/10 = 2$

hence $\theta = 63.4^o$, and therefore : $\beta_o = 180^o + \theta + \Delta = 180^o + 63.4^o + \Delta$
$= 243.4^o + \Delta$
as a starting value for iteration :
$\beta_o = 246^o$ (By choosing $\Delta = 2.6^o$) :

Estimated β		
β (degrees)	β (rad)	$\sin(\beta - \theta) + \sin\theta\ e^{-\cot\theta\ .\ \beta}$
246	4.294	- 0.033
248^o	4.33	0.023
250^o	4.363	- 0.014
249^o	4.316	0.0044
249.25^o	4.350	- 0.003

By intuition : $\beta = 249.25^o$. By intuition

(c)The average DC output voltage is :

$$V_{dc} = \frac{1}{2\pi}\int_0^{\beta} v_s\,(\omega t)d(\omega t) = \frac{1}{2\pi}\int_0^{\beta} V_m sin\omega t\ d(\omega t)$$

$$V_{dc} = \frac{V_m}{2\pi}\Big[-\cos\omega t\Big\{_0^{\beta}\Big] = \frac{V_m}{2\pi}[-\cos\beta + \cos 0] = \frac{V_m}{2\pi}[1 - \cos\beta]$$

Where V_m=300 V

Hence

$$V_{dc} = \frac{300}{2\pi}[1 - \cos 249.25] = 64.694\ V$$

(c) Angle of maximum current can be found as ,

The equation of the current (3.22) is

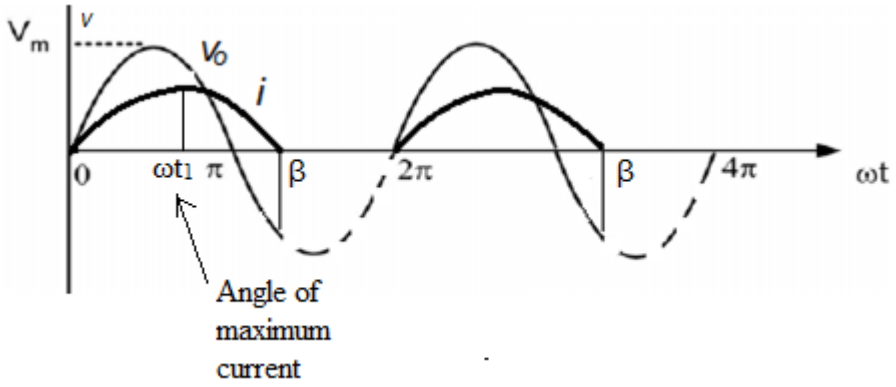
$$i(\omega t) = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}[\sin(\omega t - \theta) + sin\theta e^{-cot\theta\ .\omega t}]$$

To find the maximum value of the current $\frac{di(\omega t)}{dt} = 0$

$$\frac{di(\omega t)}{dt} = \frac{V_m}{\sqrt{10^2 + (20)^2}}[\omega\cos(\omega t - 63.4) - \omega sin\ 63.4e^{-cot63.4\ .\omega t}] = 0$$

$$\frac{di(\omega t)}{dt} = [\cos(\omega t - 63.4) - \sin\ 63.4e^{-cot63.4\ .\omega t}] = 0$$

Maximum current occurs at ωt= ωt_1 as shown in the following figure:



Hence,

$$\cos(\omega t_1 - 63.4) = sin\ 63.4e^{-cot63.4\ \omega t_1}$$

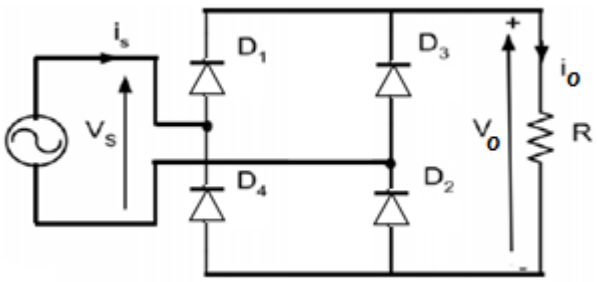
Solve this equation numerically to find ωt_1.

LECTURE .4 : Single-phase full-wave uncontrolled rectifiers

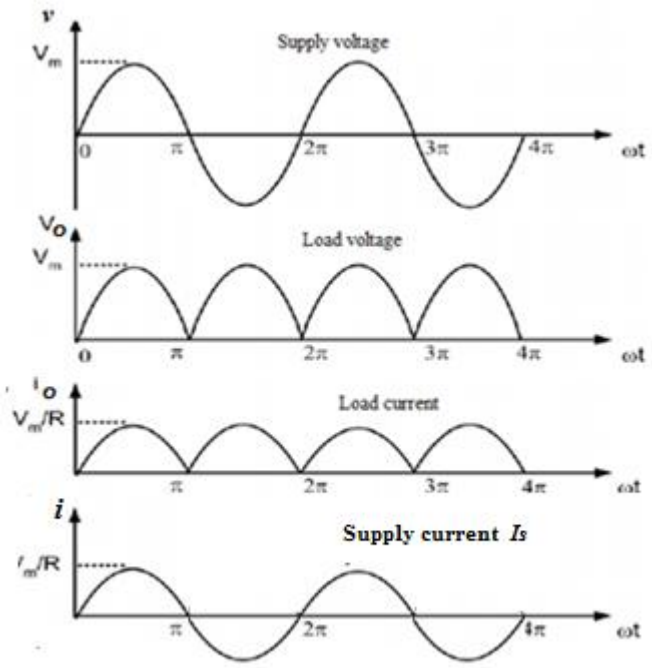
4.1- Case of Resistive Load

Fig.4.1 (a) presents the circuit connection for a single-phase full-wave, bridge rectifier loaded with a resistive load. It is sometimes referred to as the full-wave bridge rectifier. For this configuration, two diodes always conducting during the same interval to provide a closed loop for the current. D_1 and D_2 conduct whenever the supply voltage (v_S) is positive while D_3 and D_4 conduct whenever the supply voltage (v_S) is negative as illustrated by Fig .4.1 (b).

Since the load is a resistive load. Then, the load current will have the same waveform as the load voltage, $i_o = \frac{v_o}{R}$.



(a) Circuit.



(b) Waveforms

Fig.4.1 Single-phase full-wave rectifier with resistive load.

The average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m sin\omega t \, d\omega t = \frac{V_m}{\pi}\,(-cos\pi + cos0^o) = \frac{2V_m}{\pi} \qquad (4.1)$$

The value of (4.1) is seen to be twice the corresponding value of the half – wave rectifier given in eq.(3.3) - lecture .3 .

The average value of the load current I_{dc} is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} \qquad (4.2)$$

$$P_{dc} = V_{dc}I_{dc} = \frac{V_{dc}^2}{R} \quad (4.3)$$

The rms value of the load voltage V_{rms} can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v_s^2\,(\omega t)d\omega t}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m sin\omega t)^2\, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{\pi}\int_0^{\pi}\frac{1}{2}(1 - cos2\omega t)d\omega t} = \frac{V_m}{\sqrt{2}} \quad (4.4)$$

The value of V_{orms} is seen to be √2 times the corresponding value of the half – wave rectifier given in eq. (3.7) - lecture 3. It should be noted that V_{orms} has the same rms value as a sinusoidal voltage of same maximum value . This means that the rms value is not affected by the polarity of the waveform.

The rms value of the load current I_{orms} is ,

$$I_{orms} = \frac{V_{orms}}{R} = \frac{V_m}{\sqrt{2}R} \quad (4.5)$$

The output a.c. power is given by:

$$P_{ac} = V_{orms}I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.707V_m)^2}{R} \quad (4.6)$$

The PRV for any diode in this configuration is (V_m) .

The load voltage ripple factor is

$$RF = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = 0.48$$

Which is smaller than the corresponding *RF* value of the half – wave rectifier (see example 3.1 in lecture 3).

The supply current I_s is a pure sinusoidal as shown in Fig.4.1 (b). This is because the application of sinusoidal voltage to the resistive load causes sinusoidal supply current which is in – phase with the voltage. The average and rms values of the supply current are,

$$I_{s(av)} = \frac{1}{2\pi}\int_0^{2\pi} V_m sin\omega t\, d\omega t = 0 \quad (4.7)$$

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(\frac{V_m}{R}sin\omega t)^2\, d\omega t} = \frac{V_m}{\sqrt{2}R} \qquad (4.8)$$

If we neglect the voltage drops across the diodes, the input power will be equal to the output power and therefore the power factor of the circuit will be unity.

4.2 Single-phase Full-Wave Bi-phase (Center tapped) Uncontrolled Rectifier with R- Load

This rectifier is shown in Fig.4.2. The waveforms and analysis are same as for the bridge rectifier of Fig.4.1.

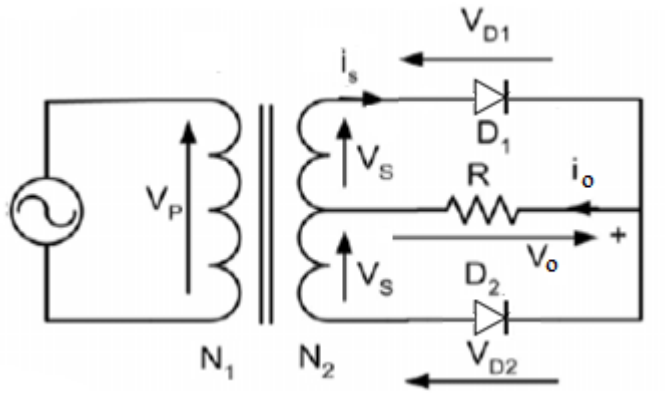


Fig.4.2 Bi – phase (center- tapped) full wave uncontrolled rectifier.

Example 4.1

For the single - phase, full-wave, uncontrolled rectifier show in Fig.2.8, the supply voltage it 110V, 50Hz .the load resistor is 25Ω.calculate:

(a)The average value of the output voltage and current.

(b)The rms value of the output voltage and current.

(c) The dc power consumed by the load (P_{dc}) and the average value of the power delivered to the load (P_{ac}).

Sketch the appropriate voltage and current waveforms.

Solution

(a) The average value of the load voltage V_{dc}

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t = \frac{2V_m}{\pi} = \frac{2\sqrt{2}x110}{\pi} = 99.035 \text{ V}$$

The average value of the load current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{99.035}{25} = 3.961 \text{ A}$$

(b) the rms value of the load voltage V_{orms} :

$$V_{orms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m \sin \omega t)^2 \, d\omega t}$$

$$\frac{V_m}{\sqrt{2}} = \frac{\sqrt{2}x110}{\sqrt{2}} = 110 \text{ V}$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{110}{25} = 4.4 \text{ A}$$

(c) The dc power,

$$P_{dc} = \frac{V_{dc}^2}{R} = \frac{(99.035)^2}{25} = 392.31 \text{ W}$$

$$P_{ac} = \frac{V_{orms}^2}{R} = \frac{(110)^2}{25} = 484 \text{ W}$$

The voltage and current waveforms are shown in Fig.4.3.

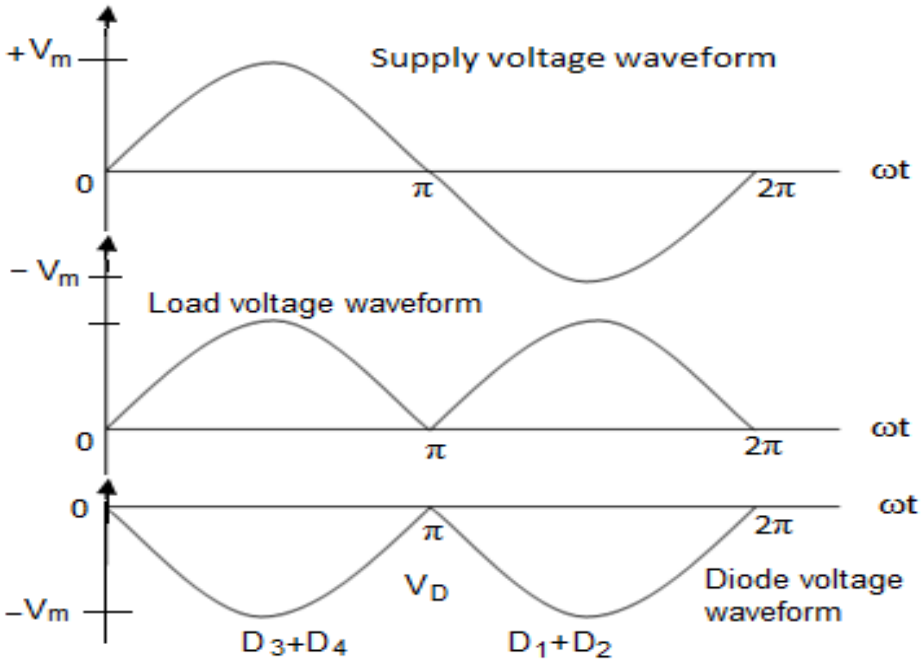


Fig.4.3 voltage and current waveforms.

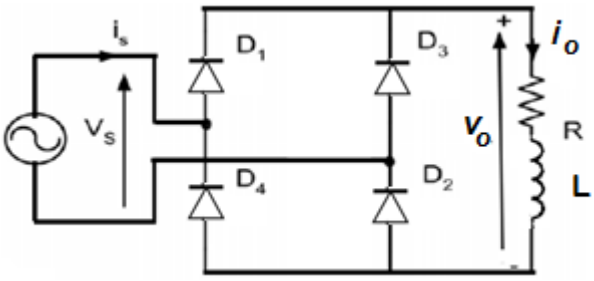
4.3 Single-phase full-wave uncontrolled rectifier loaded with highly inductive load

Fig. 4.4 (a) presents the circuit connection for a single-phase, full-wave, rectifier loaded with a highly inductive load. Highly inductive loads are basically *R-L* loads where $L >>> R$. Therefore, the load time constant $\tau = L/R$ is very high and can be considered infinity. Consequently, the load current is assumed constant. For one

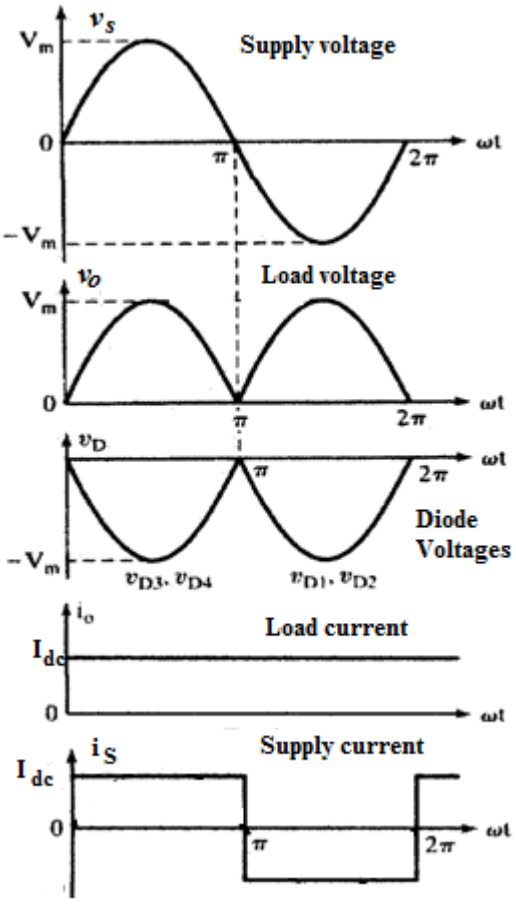
total period of operation of this circuit, the corresponding waveforms are shown in Fig. 4.4 (b).

During the conduction of D_1 and D_2 simultaneously the supply voltage appears directly across the load so that the load voltage v_o (ωt) remains the form shown in Fig.4.1(b) (same as the case of resistive load). Hence, the average value of the load voltage V_{dc} can be calculated as follows:

$$V_{dc} = \frac{1}{\pi}\int_0^{\pi} V_m sin\omega t \, d\omega t = \frac{V_m}{\pi}\,(-cos\pi + cos0^o) = \frac{2V_m}{\pi} \qquad (4.9)$$



(a) Circuit.



(b) Waveforms.

Fig.4.4 Single -phase full-wave rectifier loaded with highly inductive load.

Since the load is a highly inductive load. Then, the load current is considered constant (ripple free current) and its average value is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R} \qquad (4.10)$$

and the d.c. power is

$$P_{dc} = V_{dc} I_{dc} = \frac{V_{dc}^2}{R} \quad (4.11)$$

The rms value of the load voltage V_{rms} can be calculated as follows:

$$V_{orms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} (V_m sin\omega t)^2 \, d\omega t}$$

$$= \sqrt{\frac{(V_m)^2}{\pi}\int_0^{\pi} \frac{1}{2}(1 - cos2\omega t) d\omega t} = \frac{V_m}{\sqrt{2}} \quad (4.12)$$

Therefore the rms value of the load current I_{orms} is

$$I_{orms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2}R} \quad (4.13)$$

Since the load current is constant over the studied period, therefore the rms value of the load current I_{orms} is

$$I_{orms} = I_{dc} = I_a \quad (4.14)$$

The output AC power is given by:

$$P_{ac} = V_{orms} I_{orms} = \frac{V_{orms}^2}{R} = \frac{(0.707 V_m)^2}{R} \quad (4.15)$$

Problem 4.1

A single-phase full-wave uncontrolled bridge rectifier is supplying a highly inductive load (L/R ratio is very large), the load current is assumed to be smooth and ripple –free. If the supply voltage is 220V, 50Hz, and the inductor load resistance R=22 Ω, Calculate:

(a)The average output voltage V_{dc} and current I_{dc}.

(b) The rms value of the output voltage V_{orms} and current I_{orms} .

(c) The rms value of the diode current I_{Drms} and the PRV of each diode.

Problem 2.2 Repeat problem (4.1) with Bi-phase rectifier type and compare your results.

5.Poly- phase uncontrolled rectification

For most industrial applications poly – phase rectifier circuits are use. The circuit employed may gives either half – wave or full – wave, controlled or uncontrolled rectifier circuits.

1. Three –Phase Half -Wave Uncontrolled Rectifier

Fig.5.1 shows a 3-phase half-wave uncontrolled rectifier with resistive load. The rectifier is fed from an ideal 3 – phase supply through delta –star 3- phase transformer. The principle of operation of this convertor can be explained as follows:

- Diode 1 which has a more positive voltage at its anode conducts for the period from π/6 to 5 π / 6. In this period D2 and D3 are off. The neutral wire provides a return path to the load current.
- Similarly, diode 2, and 3, whichever has more positive voltage at its cathode conducts.
- The conduction pattern is: **D1 ,D2, D3.**

The output voltage and current waveforms are shown in Fig.5.2.

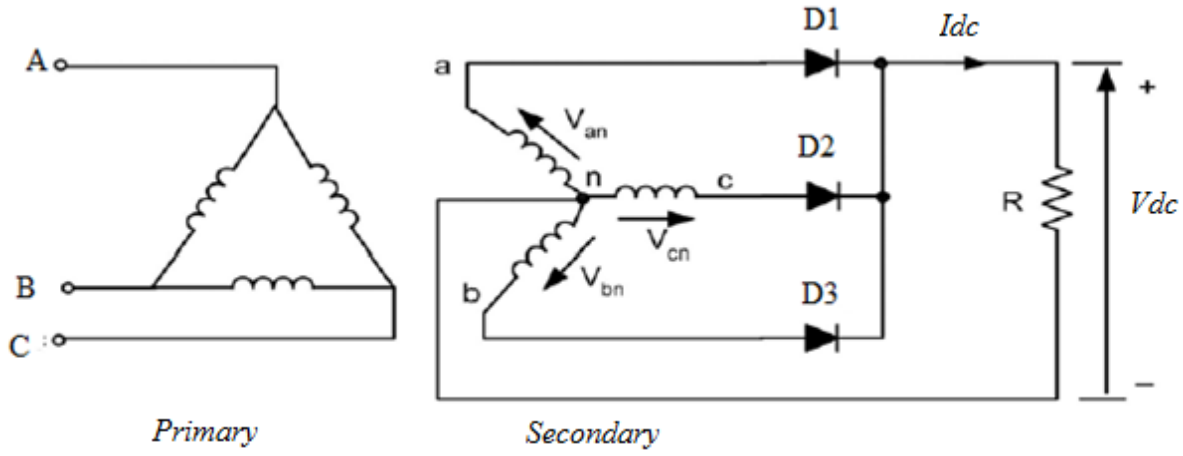


Fig.5.1 A 3-phase half-wave uncontrolled rectifier with resistive load.

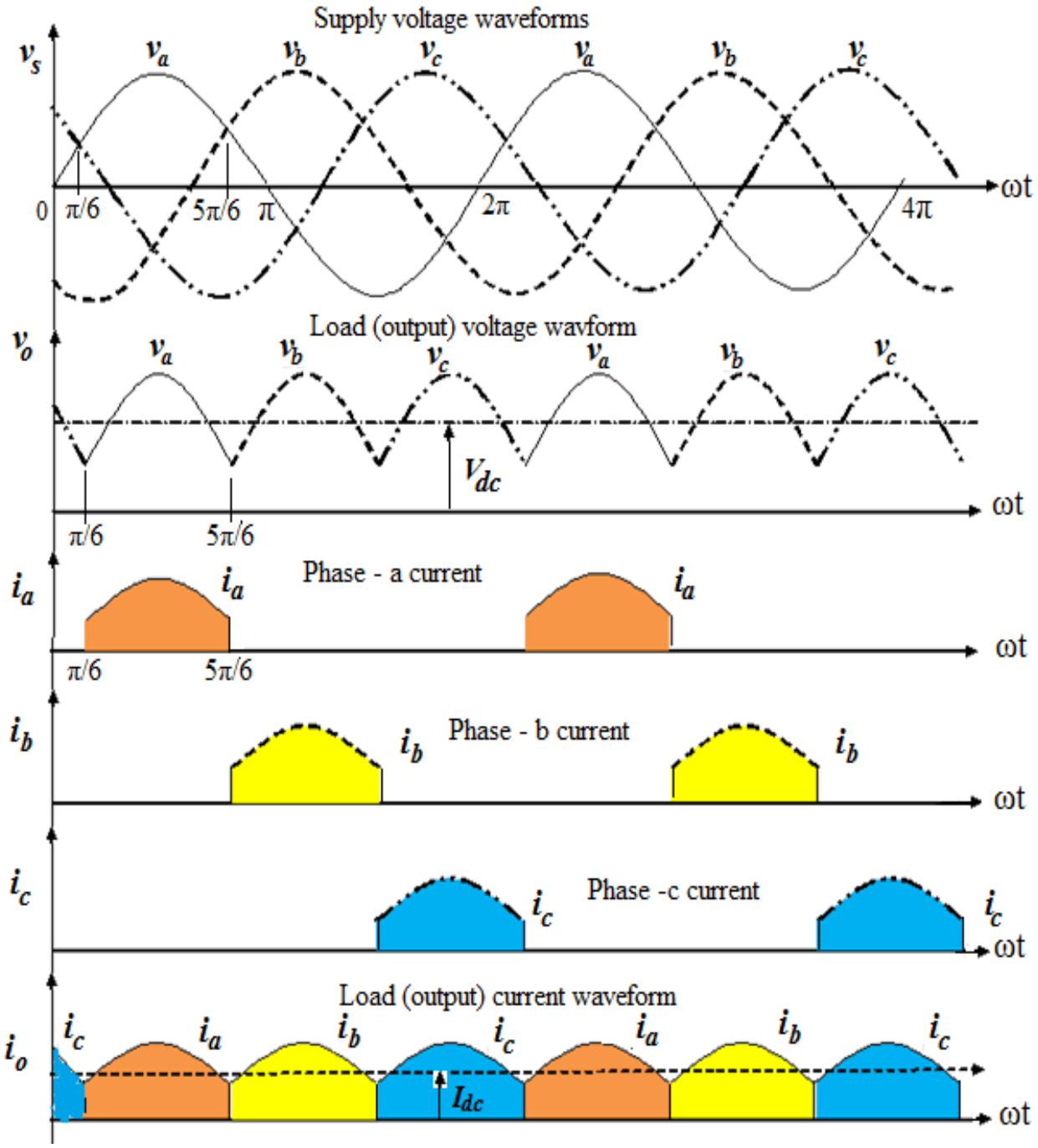
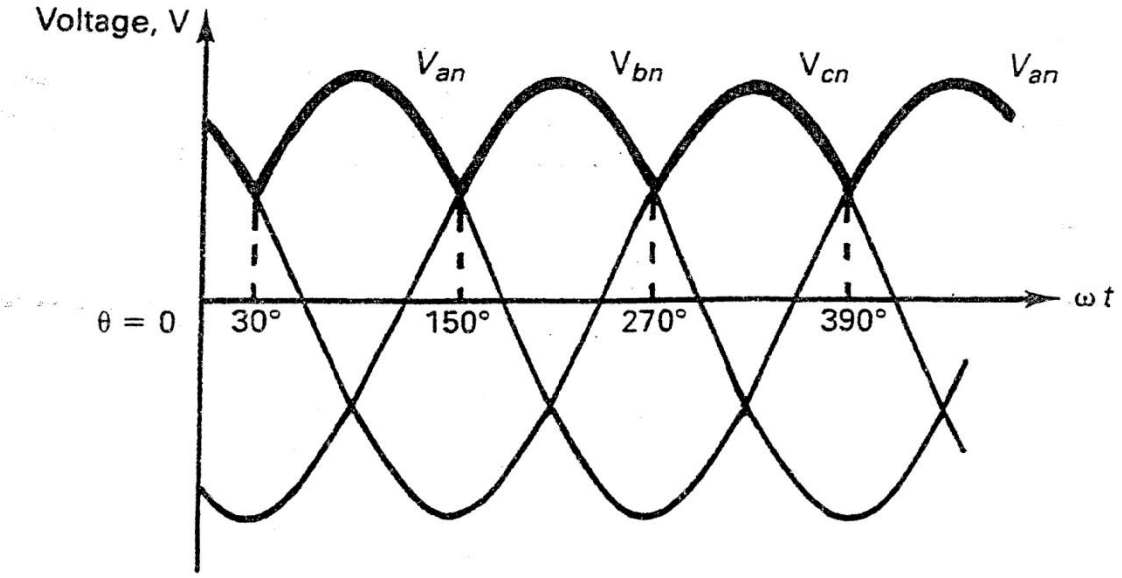


Fig .5.2 Load voltage and current waveforms for the 3 – phase hall – wave uncontrolled rectifier.

Analytical properties of the output voltage waveform

The average value of the voltage waveform in Fig.5.3 can be found as follows:



Let $v_{an} = V_m \sin \omega t$

$v_{bn} = V_m \sin(\omega t - 2 \pi / 3)$

$v_{cn} = V_m \sin(\omega t - 4 \pi / 3)$

The average value of the load voltage wave is

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{\pi/6}^{5\pi/6} V_m \ sin\omega t \ d\omega t = \frac{3V_m}{2\pi} \left[-cos\omega t\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{3V_m}{2\pi}\left[-(\cos\frac{5\pi}{6} - cos\frac{\pi}{6})\right] = \frac{3V_m}{2\pi}\left[-(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2})\right] = \frac{3\sqrt{3}V_m}{2\pi}$$

The load current I_{dc} is:

$$I_{dc} = \frac{3\sqrt{3}.Vm}{2\pi R}$$

Note that the secondary windings of the supply transformer carry unidirectional currents, which leads dc magnetization of the transformer core. This implies that the transformer cores have dc flux, so that for the same ac voltage and hence flux swing, it must have larger core size than is necessary. This problem of dc magnetization is avoided using bridge rectifier circuit.

2.Three –Phase Full-Wave Uncontrolled Bridge Rectifier

Fig.5.4 shows a 3-phase full-wave uncontrolled bridge rectifier with resistive load. The rectifier is fed from an ideal 3 – phase supply through delta –star 3-phase transformer. The principle of operation of this convertor can be explained as follows:

Each three-phase line connects between pair of diodes .One to route power to positive (+) side of load, and other to route power to negative (-) side of load.

- Diode 1, 3 and 5, whichever has a more positive voltage at its anode conducts.
- Similarly, diode 2, 4 and 6, whichever has more negative voltage at its cathode return the load current.
- The conduction pattern is: **12-16-36-34-54-52-12.**
- Each diode conducts for 120˚ in each supply cycle as shown in Fig.5.5.

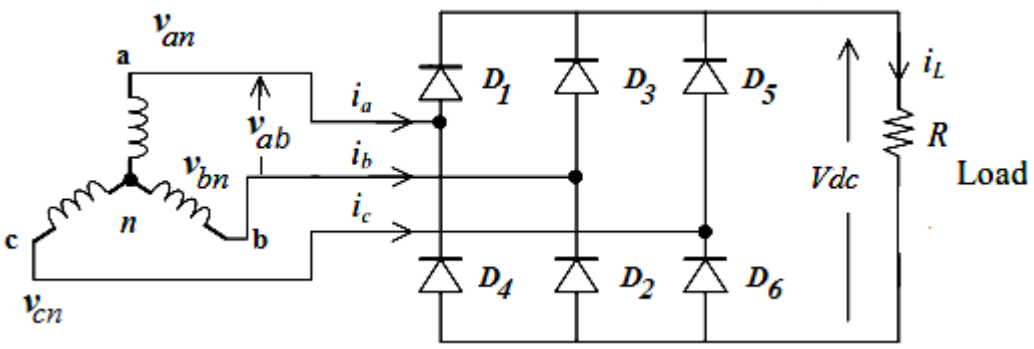


Fig.5.4 The rectifier

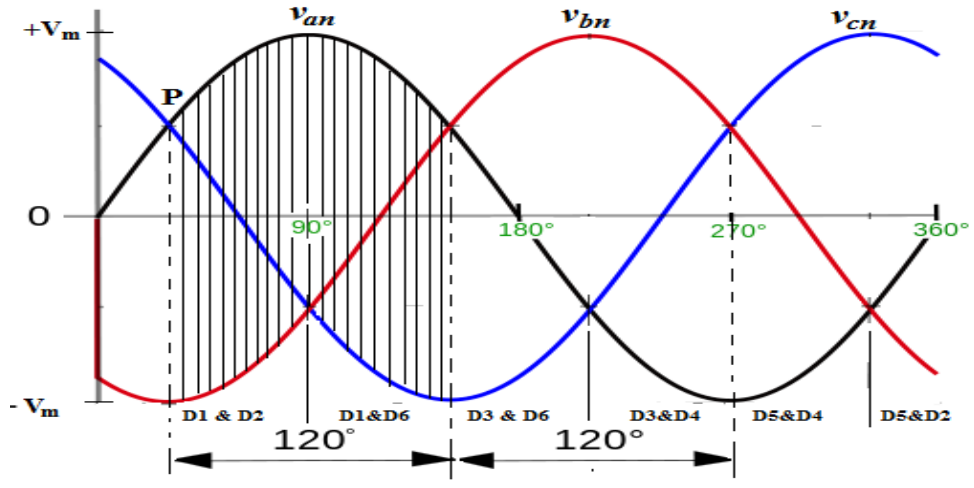


Fig.5.5 Each diode conducts for 120˚.

The output voltage is the instantaneous difference between two appropriate phases at each instant as depicted in Fig.5.6, and the resultant dc output voltage wave is shown in Fig.5.7.

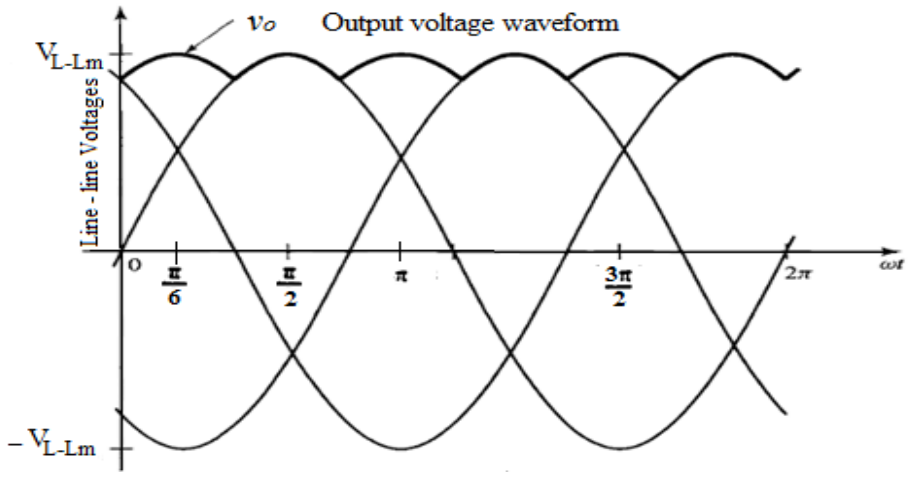


Fig.5.6 The line to line supply voltage and the output voltage waveforms.

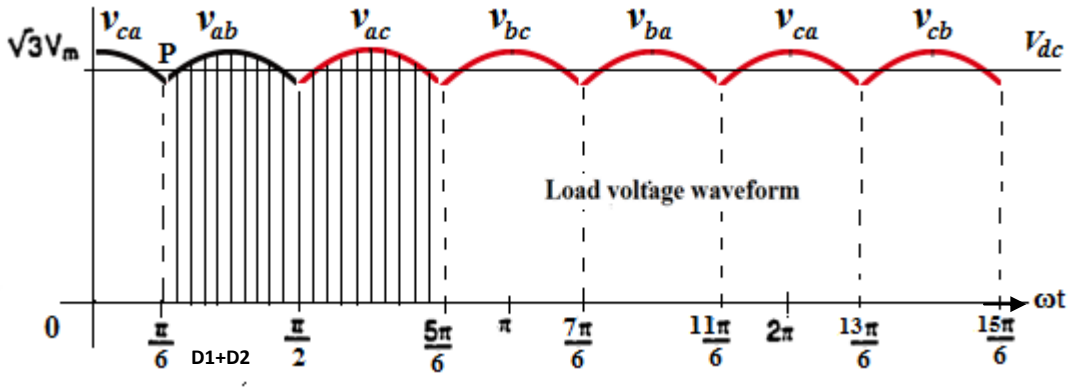


Fig.5.7 The dc output voltage waveform.

The average output voltage:

To find the average voltage V_{dc} on the load, assume that the line –to – line voltages are represented by the following equations,

$$v_{ab} = v_{an} - v_{bn} = V_m \sin(\omega t) - V_m \sin(\omega t - 2\pi/3),$$

hence

$$v_{ab}(\omega t) = \sqrt{3}\, V_m sin\left(\omega t + \frac{\pi}{6}\right)$$

Similarly,

$$v_{bc}(\omega t) = \sqrt{3}\ V_m sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_{ca}(\omega t) = \sqrt{3}\ V_m sin\left(\omega t + \frac{7\pi}{6}\right)$$

Hence: by integrating over 1/6 of a cycle,

$$V_{dc} = \frac{3}{\pi}\int_{\pi/6}^{\pi/2} v_{ab}(\omega t)\, d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m sin\left(\omega t + \frac{\pi}{6}\right) d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m(\sin\omega t \cos\frac{\pi}{6} + \cos\omega t \sin\frac{\pi}{6})d\omega t$$

$$= \frac{3}{\pi}\int_{\pi/6}^{\pi/2} \sqrt{3}V_m\left(\sin\omega t\ \frac{\sqrt{3}}{2} + \cos\omega t\frac{1}{2}\right) d\omega t$$

$$= \frac{3\sqrt{3}.Vm}{\pi}[-\cos(\pi/2) + \cos(\pi/6)]x\frac{\sqrt{3}}{2} + [\sin(\pi/2) - \sin(\pi/6)]x\frac{1}{2}$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[0 + \frac{\sqrt{3}}{2}\right]x\frac{\sqrt{3}}{2} + \left[1 - \frac{1}{2}\right]x\frac{1}{2}$$

$$= \frac{3\sqrt{3}V_m}{\pi}\left[\frac{3}{4} + \frac{1}{4}\right] = \frac{3\sqrt{3}V_m}{\pi}$$

The load current I_{dc} is:

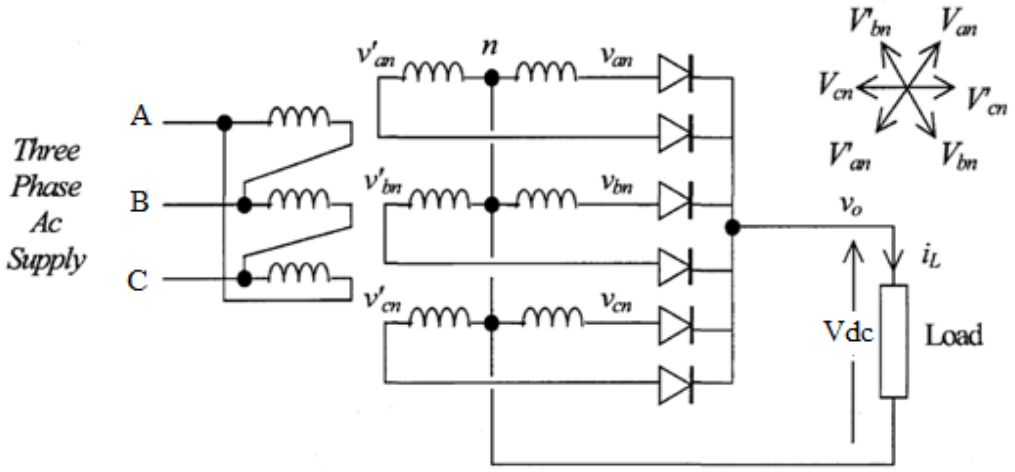
$$I_{dc} = \frac{3\sqrt{3}V_m}{\pi R}$$

The average power is:

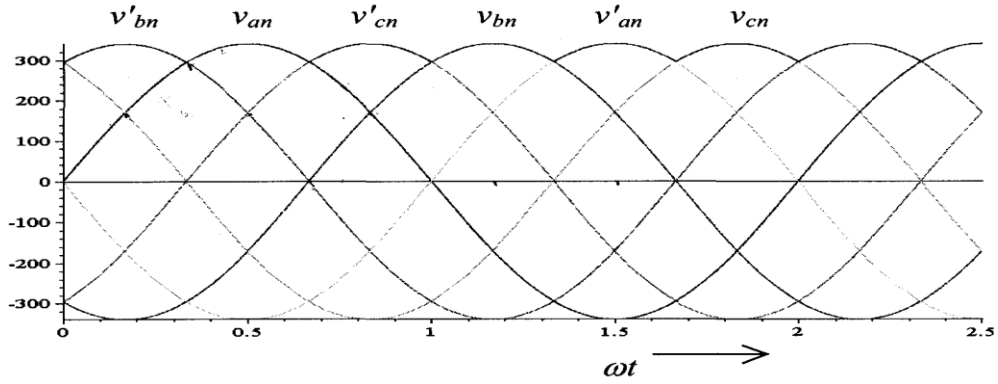
$$P_{dc} = V_{dc}I_{dc} = \frac{3\sqrt{3}V_m}{\pi}x\ \frac{3\sqrt{3}V_m}{\pi R} = \frac{27\ V_m}{\pi R}$$

3. Six –phase (hexa-phase) uncontrolled rectifier

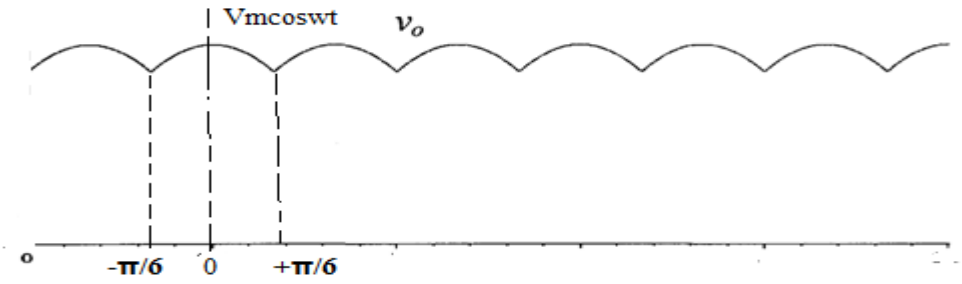
To get more smooth output voltage waveform, a six –phase supply is obtained from a three-phase system using transformer with centre tapped secondary winding as shown in the Fig.5.8 below:



(a)



(b)



(c)

Fig.5.8 Six – phase rectifier (a) Circuit, (b) Input voltages, (c) output voltage

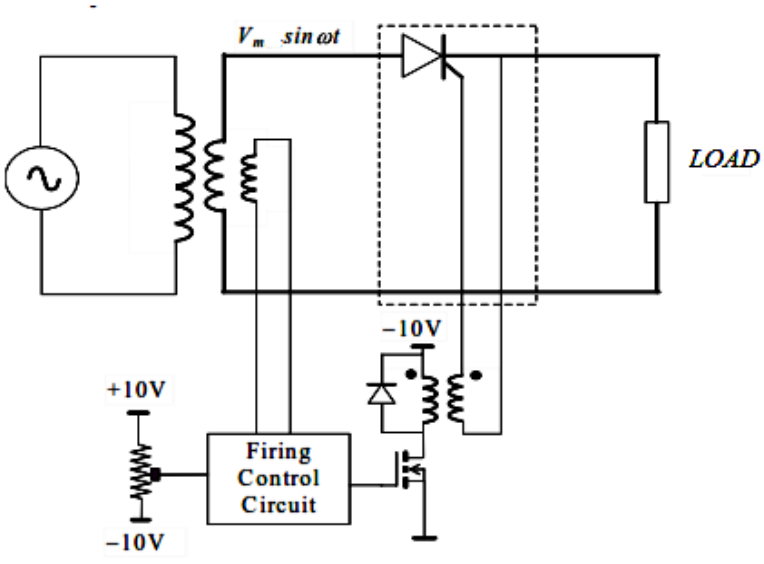
The average output voltage is: $\mathrm{V_{dc}} = \frac{3}{\pi}\int_{-\pi/6}^{\pi/6} V_m \cos \omega t \, d\omega t = \frac{3V_m}{\pi}$

6. Controlled Rectifications

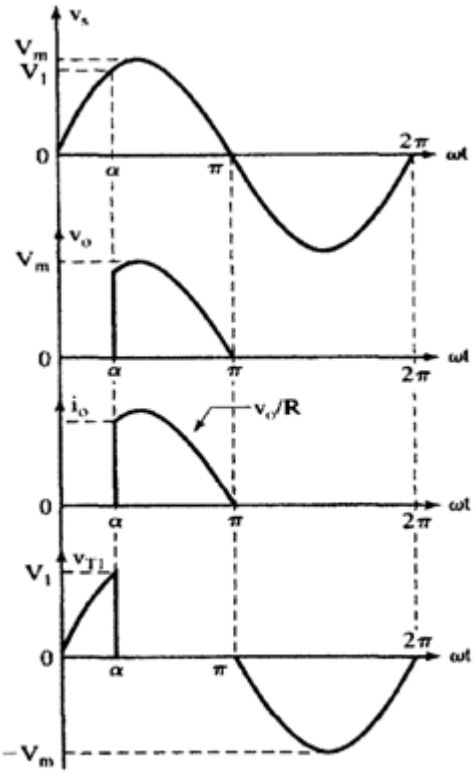
In this type, the generated DC power is controllable and variable. They usually use SCRs as their power switches. For fast switching operation, MOSFETs and IGBTs are used. The following subsections deal with the basic operation of some examples of controlled rectifiers single-phase half-wave controlled rectifier loaded with resistive load.

1. Single-phase, half-wave, controlled rectifier loaded with a resistive load

Fig. 6.1(a) shows the basic circuit for a single-phase, half-wave, controlled rectifier loaded with a resistive load. For this configuration, the thyristor will conducts when triggered using gate pulses provided that the supply voltage (v_S) is positive.The thyristor is fired at $\omega t=\alpha$ and the input voltage appears across the load. At $\omega t = \pi$, T_1 is reverse-biased by the negative supply voltage and is turned off. 'α' is termed as the delay or firing angle. The converter is said to operate in the first quadrant. This converter is not normally used in the industrial application because of its high ripple current and low ripple frequency. The waveforms for one total period of operation of this circuit are shown in Fig. 6.1(b).



(a) Circuit



(b) Waveforms

Fig.6.1 Single-phase half-wave controlled rectifier.

The average value of the load voltage V_{dc} can be calculated as follows

$$V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi} v_s(\omega t)\, d\omega t = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t$$

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha)$$

The average value of the load current I_{dc} is

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R}(1+\cos(\alpha))$$

Therefore, the average output voltage can vary from 0 to V_m/π and the average load current will vary from 0 to $V_m/\pi R$ when varying α from π to 0, respectively.

The rms value of the load voltage V_{rms} can be calculated as follows,

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\{v_s(\omega t)\}^2\, d\omega t} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}\{V_m \sin(\omega t)\}^2\, d\omega t}$$

$$\therefore V_{rms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

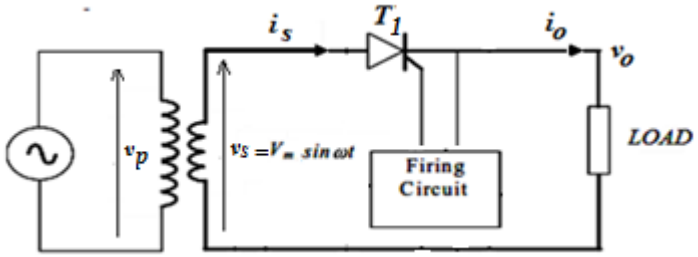
Therefore the rms value of the load current I_{rms} is

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

The PRV of the thyristor for this configuration is V_m .

Example -1 : A single- phase half - wave controlled rectifier shown in Fig.1, operates from an ideal sinusoidal source with v_s = 325 sinωt, at 50 Hz . If the load is purely resistive, and at a certain triggering α , the output d.c. voltage is V_{dc} = 95V, and the average value of the load current is 2.2 A. It is required to calculate the following:

(a) The triggering angle α
(b) Load resistance
(c) R.m.s load voltage
(d) R.m.s load current
(e) DC power
(f) The ripple factor



Solution

(a) For single-phase half-wave controlled rectifier with resistive load;

$$V_{dc} = \frac{V_m}{2\pi}(1 + cos\alpha)$$

$$95 = \frac{325}{2\pi}(1 + cos\alpha)$$

Hence α = 33.31˚ .

(b)

$$R = \frac{V_{dc}}{I_{dc}} = \frac{95}{2.2} = 43.18\Omega$$

(c) r.m.s load voltage

$$V_{orms} = \frac{V_m}{2}\sqrt{\frac{1}{\pi}(\pi - \alpha + \frac{1}{2}sin2\alpha)}$$

$$= \frac{325}{2}\sqrt{\frac{1}{\pi}(\pi - 0.58 + \frac{1}{2}sin66.62°)} = 159.34\,V$$

(d) R.m.s load current

$$I_{orms} = \frac{V_{orms}}{R} = \frac{}{43.18} = 3.7A$$

(e) The output DC power is given by:

$$P_{dc} = V_{dc}I_{dc} = \frac{V_{dc}^2}{R} = \frac{95^2}{43.18} = 209\,W$$

(f)

$$\text{Ripple factor} = \frac{\sqrt{V_{orms}^2 - V_{dc}^2}}{V_{dc}} = \frac{\sqrt{159.34^2 - 95^2}}{95} = 1.34$$

2. The Freewheeling Diode

The freewheeling diode is connected in the circuit in such away as to provide an alternative path for the decaying load current so that the thyriator current is al1owed to become zero and the thyristor is allowed to switch off.

Consider the half—wave rectifier of Fig 6.2 where FD is the freewheeling: diode. Then the supply voltage is positive, from α to π, FD is in reverse and passes no current,, so that source end load current are equal (is = i). During the negative helf—cycle, the load current i flows through the low resistance path provided by FD rather than against the negative supply voltage, so that $i_{FD} = i$, and $i_s = 0$. Hence the thyristor.

This allowed to switch off. In this part of the half— cycle, the current is driven by the energy stored in L; it decays according to the time constant of the circuit (R, L, and FD); v is very small and negative, being equal to the voltage drop across FD.

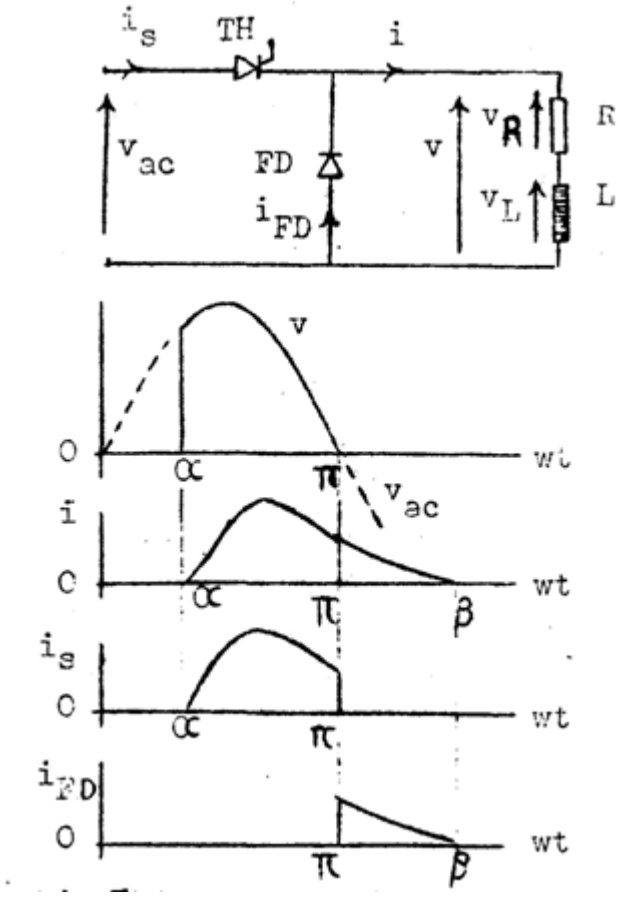


Fig.6.2

3. Single-phase controlled converter circuits

3.1 Single-phase, fully-controlled bridge rectifier (p = 2)

Single-phase, fully-controlled full- wave rectifier bridge is shown in Fig.6.3.In this this circuit, two thyristors must triggered simultaneously to permit current to flow. For example, with the instantaneous polarity indicated in Fig.6.3, T1 and T4 must be triggered, while in reverse, T3 and T2 must triggered at the same time. The output voltage waveform is shown in Fig.6.4 for the case of resistive load.

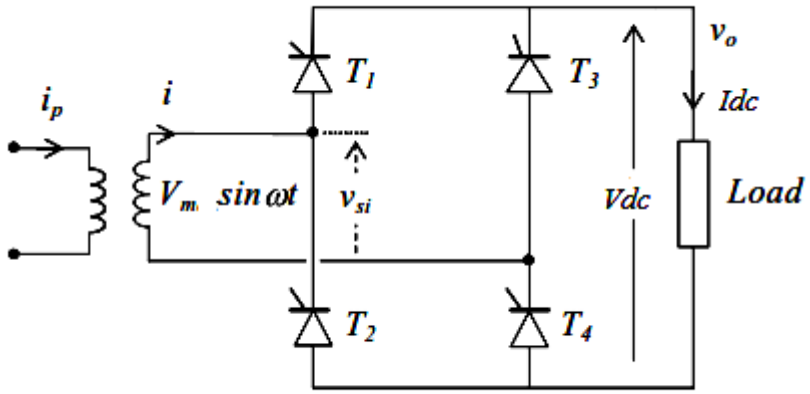


Fig. 6.3 Single-phase fully-controlled full- wave Rectifier Bridge

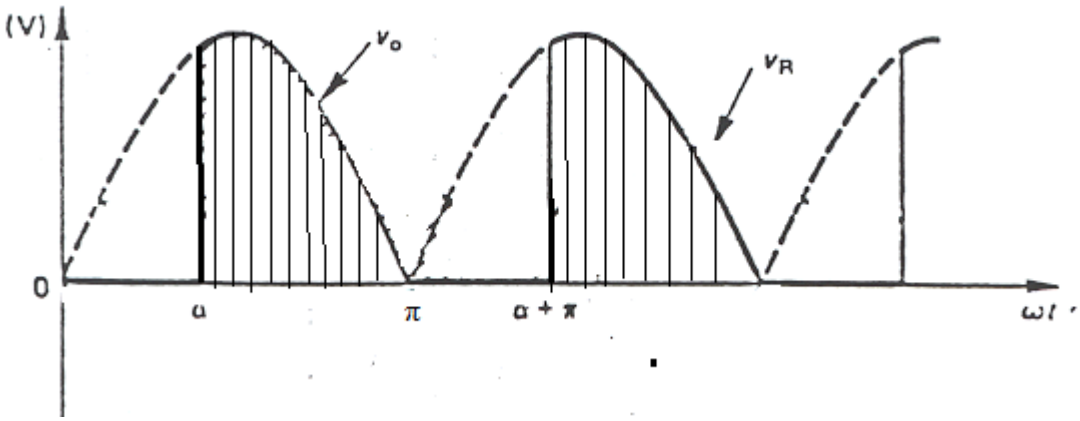


Fig.6.4 Output voltage waveform

- **Operation of the converter with Resistive load**

The dc output voltage of the converter with resistive load is given by

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin \omega t \, d\omega t$$

$$= \frac{V_m}{\pi}(1+\cos\alpha)$$

$$\text{RMS output vol tage} = V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}(V_m \sin \omega t)^2 d\omega t}$$

$$= V_m\sqrt{\frac{\pi - \alpha + \frac{1}{2}\sin 2\alpha}{2\pi}}$$

- **Operation of the converter with R – L load**

(i) Case of R - L load with small L / R ratio : <u>Discontinuous load current</u> .

In this case the current will be discontinuous as shown in Fig.6.5.

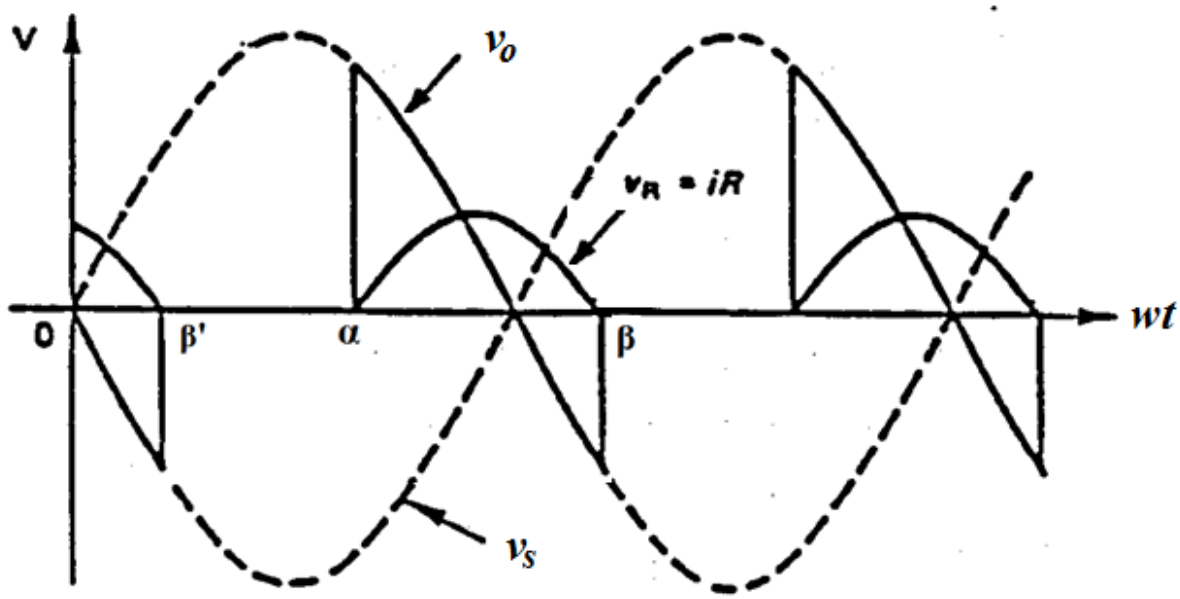


Fig.6.5 Discontinuous current operation.

For $\alpha \le \omega t \le \beta$, the circuit equation is given by :-

$$\text{Average output vol tage} = V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m \sin \omega t \, d\omega t$$

$$= \frac{V_m}{\pi}(\cos\alpha - \cos\beta)$$

(ii) Case of R - L load with large L / R ratio: <u>Continuous load current</u> .

Under these conditions, a thyristor is still conducting when another is forward-biased and is turned on. The first device is instantaneously reverse-biased by the second device which has been turned on. The first device is commutated and load current is instantaneously transferred on the incoming device . In this case the current is continuous as shown in Fig.6.6.

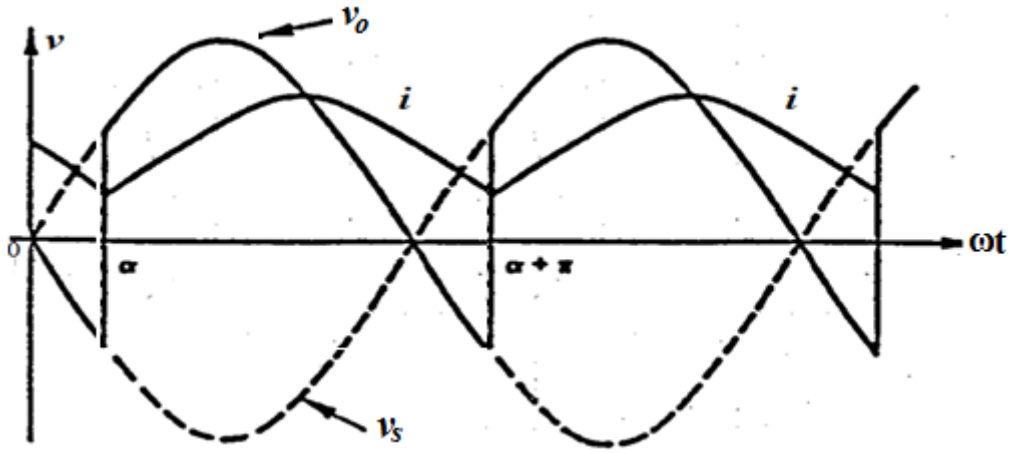


Fig. 6.6

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

Under all delay angle condition, the average current is given by:

$$I_{dc} = \frac{V_{dc}}{R}$$

From the above voltage equation, if the firing angle is greater than 90°, the average voltage can be negative .Thus if the firing angle is suddenly increased to 170° , a large negative voltage will be applied to the load and the power is fed back to the supply. This process is known as 'INVERSION'.
The graph shown in Fig. 6.7 gives the relation between the firing angle and the output voltage in p.u. for the two modes of operation (continuous and discontinuous) for full-wave single- phase rectifier.

Rule – of- thumb: To find roughly the current is continuous or discontinuous:

If $\pi + \alpha < \beta$ The current is continuous
If $\pi + \alpha > \beta$ The current is discontinuous

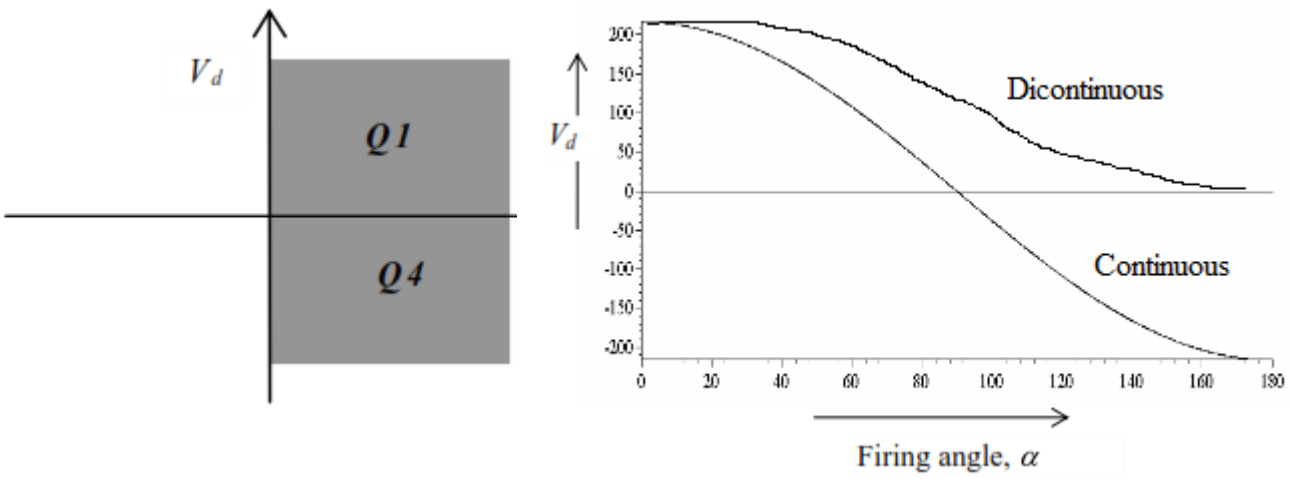


Fig. 6.7

Example: A single-phase fully controlled full-wave rectifier shown in Fig.8 has a source of 220 V r.m.s at 50 Hz, and is feeding a load R = 15 Ω and L = 20 mH. The firing angle $\alpha = 30°$ and the current extinction angle $\beta = 225$.

(a) Specify whether the current is continuous or discontinuous.
(b) Sketch the appropriate load voltage and load current waveforms
(c) Determine the average load voltage and current.
(d) Determine the r.m.s load voltage and current.
(e) Determine the a.c and d.c powers absorbed by the load.
(f) If thyristor T_3 fails to open circuit, what will be the average output voltage?

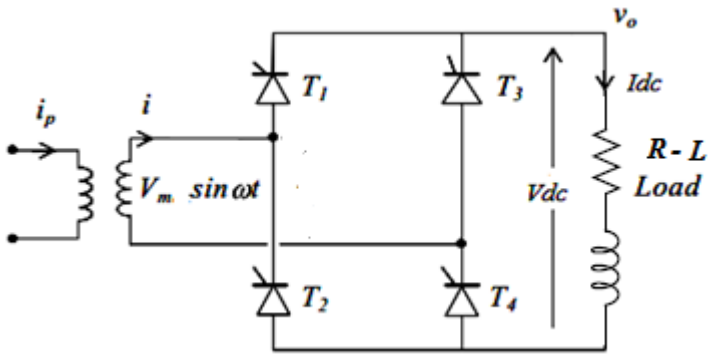


Fig.8

Solution

(a) Since

$\pi+\propto= 180° + 30° = 210° < \beta$ then the current is continous

(b) The load voltage v_o and load current i waveforms are as in figure 6.9

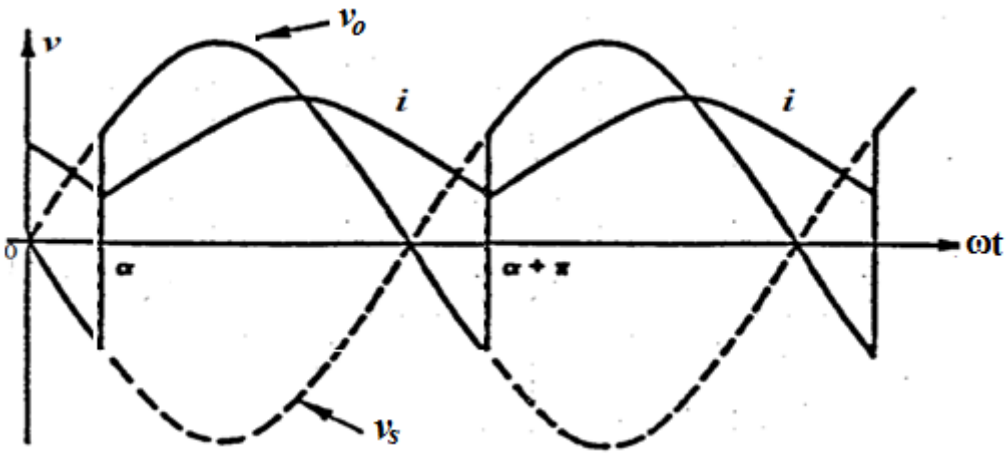


Fig.6.9 Same as Fig.6.6.

(c)

$$V_m = 220 \times \sqrt{2} = 311.08\ \text{V}$$

$$V_{dc} = \frac{1}{\pi}\int_{\propto}^{\propto+\pi} V_m sin\omega t\, d\omega t = \frac{2\,V_m}{\pi} cos\alpha$$

$$= \frac{2 \times 311.08}{\pi}\cos 30 = 171.12\ V$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{171.12}{15} = 11.44\ A$$

(e) The rms values of the output voltage and current:

$$V_{orms} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}(\pi - \alpha + \frac{1}{2}sin2\alpha)} = \frac{220x\sqrt{2}}{\sqrt{2}}\sqrt{\frac{1}{\pi}(\pi - \frac{\pi}{6} + \frac{1}{2}sin60°)}$$
$$= 216.788\ \text{V}$$

$$I_{orms} = \frac{V_{rms}}{Z_L} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}} = \frac{216.88}{15^2 + (2\pi x50x20x10^{-3})^2} = 13.29\ \text{A}$$

(f) The dc and ac powers are,

$$P_{dc} = V_{dc}\, I_{dc} = 171.12 \times 11.44 = 856.73\ \text{W}$$

$$P_{ac} = V_{rms}\, I_{rms} = 216.788 \times 13.29 = 2881.78\ \text{W}$$

(g) If thyristor T_3 fails to open circuit, then the cct will act as half-wave controlled rectifier

$$V_{dc} = \frac{V_m}{2\pi}(1 + cos\alpha) = 311.08 \times \frac{1}{2\pi}(1 + cos30) = 92.43V$$

(iii) Case of highly inductive load. (L > > R)

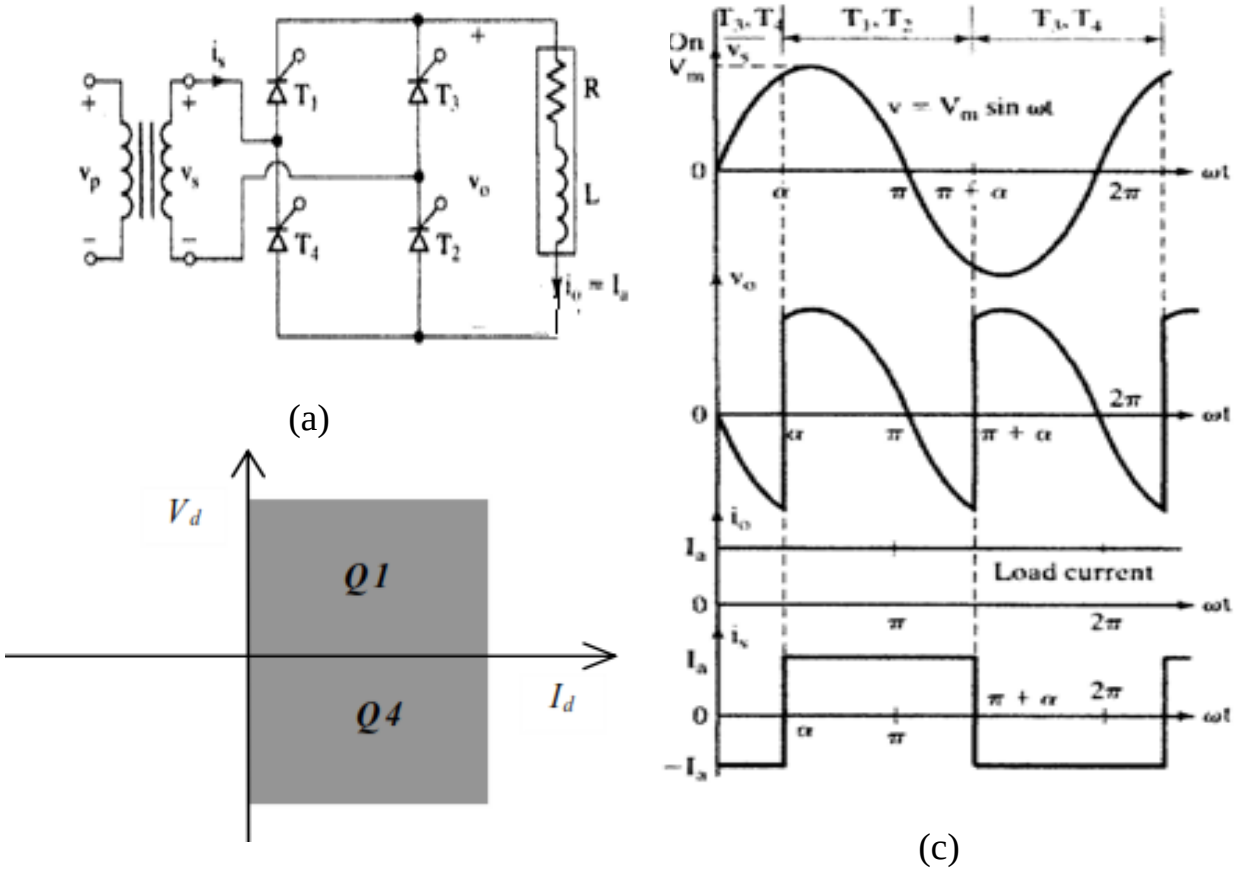
Fig. 6.8 (a) shows the circuit connection for a single-phase, full-wave, controlled rectifier loaded with a highly inductive load. For one total period of operation of this circuit, the corresponding waveforms are shown in Fig. 6,8(c) . The average value of the load voltage V_{dc} can be calculated as follows,

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} v_s(\omega t)\, d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\, d\omega t\, d\omega t$$

$$\therefore V_{dc} = \frac{2V_m}{\pi}\cos(\alpha)$$

Since the load is a highly inductive load. Then, the load current is considered constant (ripple free current) and equal to the average value of the load current I_{dc} as follows,

$$I_{dc} = I_a = \frac{V_{dc}}{R} = \frac{2V_m}{\pi R}\cos(\alpha)$$



(a)

(b)

(c)

Fig. 6.8 Single phase full-wave rectifier loaded with highly inductive load

Therefore, the average output voltage can vary from $+2V_m/\pi$ to $-2V_m/\pi$ when varying α from π to 0, respectively. Moreover, since the load voltage for this configuration can be positive or negative while the load current is always positive because the thyristors prevents a reverse current flow. Therefore, this converter operates in the first and the fourth quadrants as shown in Fig. 6.8(b).

The rms value of the load voltage V_{rms} can be calculated as follows,

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\{v_s(\omega t)\}^2\, d\omega t} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha}\{V_m \sin(\omega t)\}^2\, d\omega t} \quad = \frac{V_m}{\sqrt{2}}$$

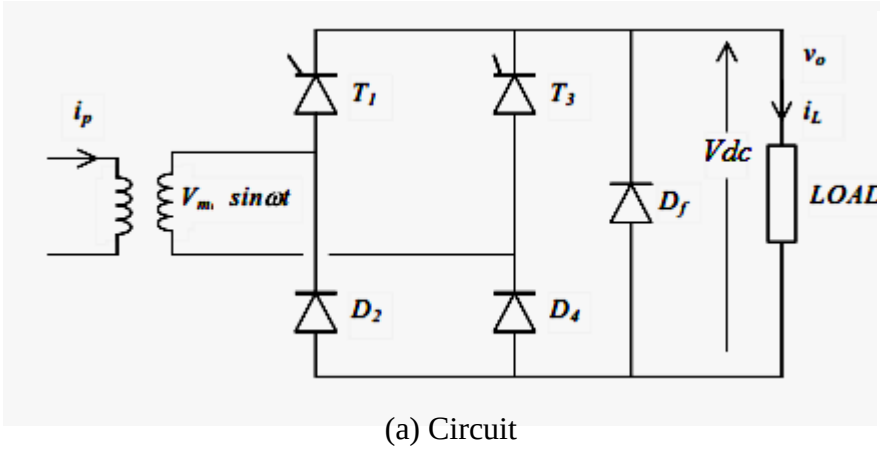
Since the load current is constant over the studied period, therefore the rms value of the load current I_{rms} is :

$$I_{rms} = I_{dc} = I_a$$

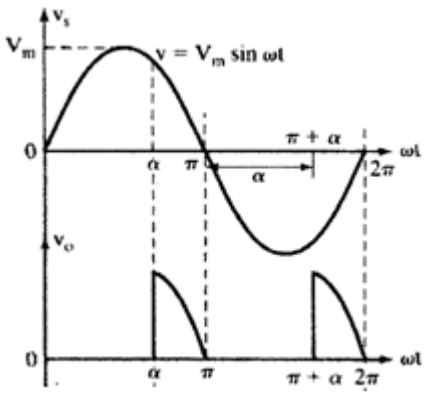
The PRV for any thyristor in this configuration is (V_m).

7. Single-phase Half – Controlled (Semiconverter) Rectifier

Fig. 7.1 (a) shows a single-phase half-controlled (semiconverter) rectifier. This configuration consists of a combination of thyristors and diodes and used to eliminate any negative voltage occurrence at the load terminals. This is because the diode D_f is always activated (forward biased) whenever the load voltage tends to be negative. For one total period of operation of this circuit, the corresponding waveforms are shown in Fig. 7.1(b).



(a) Circuit



(b) Waveform

Fig.7.1

The average value of the load voltage V_{dc} can be calculated as follows,

$$V_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} v_s(\omega t)\, d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d\omega t$$

$$\therefore V_{dc} = \frac{V_m}{\pi}(1+\cos(\alpha))$$

Therefore, the average output voltage can vary from 0 to V_m/π when varying α from π to 0 respectively.

The average value of the load current I_{dc} as follows,

$$I_{dc} = I_a = \frac{V_{dc}}{R}$$

The rms value of the load voltage V_{rms} can be calculated as follows

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\{v_s(\omega t)\}^2\, d\omega t} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\{V_m \sin(\omega t)\}^2\, d\omega t}$$

$$\therefore V_{rms} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin(2\alpha)}{2}\right)}$$

7.2 Three-phase controlled rectifier

1. Three- phase half – wave controlled rectifier (*p = 3*)

The Three- phase half – wave controlled rectifier is shown in Fig. 7.2 .As for the half – wave 3-phase uncontrolled diode rectifier, the load is connected between the converter positive terminal (cathodes of all thyristors) and the supply neutral. The diode with the highest voltage w.r.t. the neutral conducts. As the voltage of another diode becomes the highest, the load current is transferred to that device, and the previously conducting device is reverse-biased and naturally commutated. The waveforms for the supply voltage, output voltage, and load current are shown in Fig.7.3 .

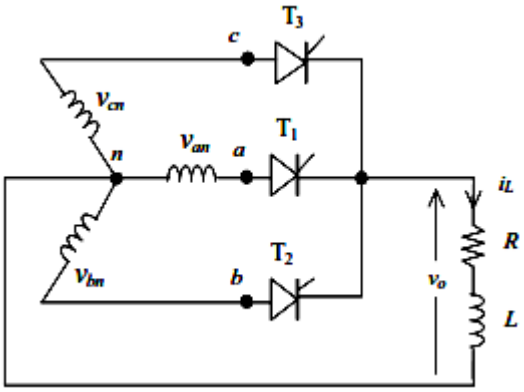


Fig.7.2 Circuit.

Note:
α is usually measured from point P in Fig.7.3.

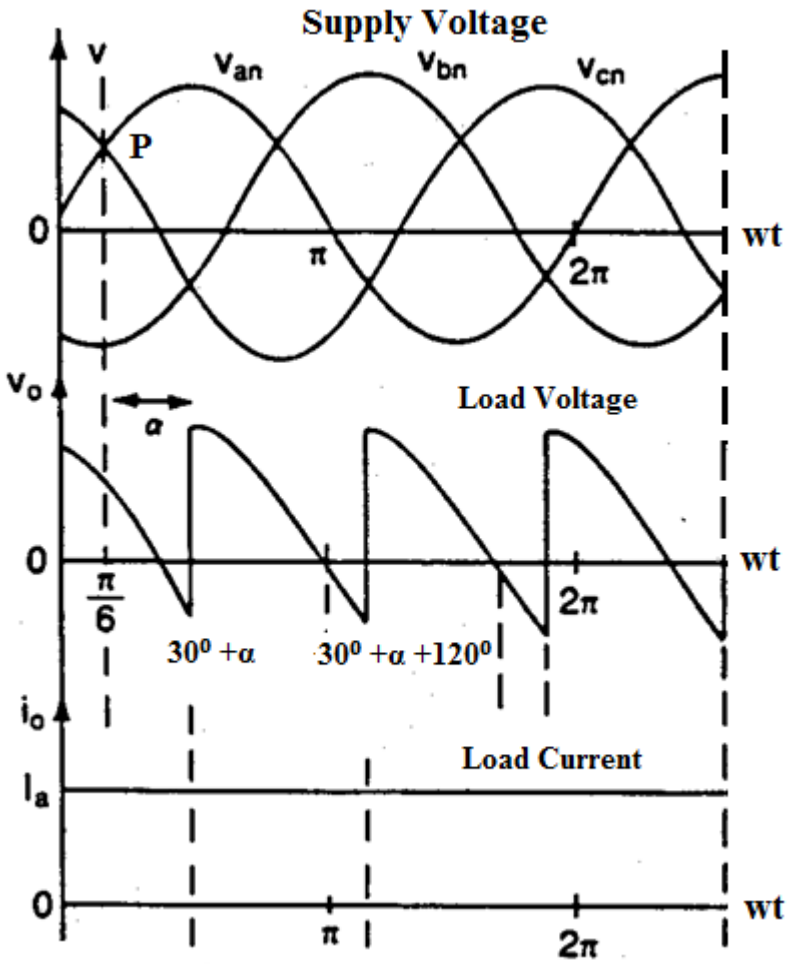


Fig.7.3 Waveforms.

The average value of the output voltage V_{dc} can be found as:

Let $v_{an} = V_m \sin \omega t$

$V_{bn} = V_m \sin(\omega t - 2 \pi / 3)$

$V_{cn} = V_m \sin(\omega t - 4 \pi / 3)$

The average value of the load voltage wave is

$$V_{dc} = \frac{1}{\frac{2\pi}{3}} \int_{30°+\alpha}^{30°+\alpha+120°} V_m \ sin\omega t \ d\omega t = \frac{3V_m}{2\pi} \ [-cos\omega t] \begin{matrix} 150° + \alpha \\ 30° + \alpha \end{matrix}$$

$$= \frac{3V_m}{2\pi}[-(\cos(150° + \alpha) - \cos\ (30° + \alpha)] = \frac{3V_m}{2\pi}\left[-\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) cos\alpha\right]$$

$$= \frac{3\sqrt{3}V_m}{2\pi} \ cos\alpha$$

The load current I_{dc} is:

$$I_{dc} = \frac{3\sqrt{3}\ V_{dc}}{2\pi R} cos\alpha$$

The operation of the 3 – phase half – wave rectifier with different values of α is illustrated in Fig.7.4.It can be seen that this converter can operate either as a rectifier or as an inverter as

For $0° < \alpha < 90°$ ⟶ Rectifier

$90° < \alpha < 180°$ ⟶ Inversion

Notes:

1. The mean output voltage is zero for $\alpha = \pi/2$. The converter is idle (no output)
2. Negative average output voltage occurs when $\alpha > \pi/2$.
3. Power inversion is possible, if a load with an e.m.f. to assist the current flow.

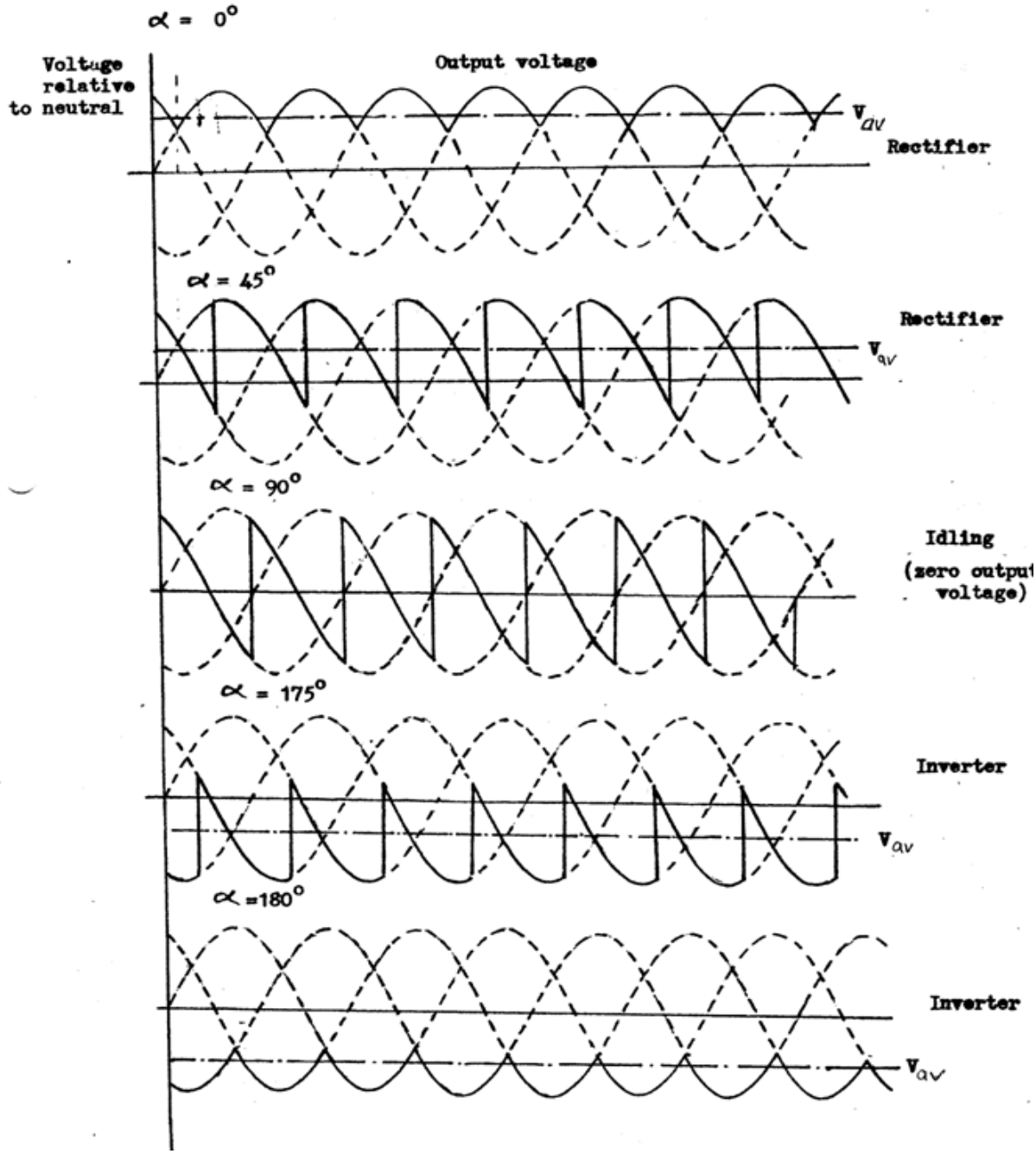


Fig.7.4 Output voltage waveform of the 3-phase half-wave rectifier for different values of firing angle α. Case of R-L load.

Example : For the three-phase half-wave fully controlled rectifier shown in Fig.7.2, the load consists of a resistance and a very large inductance. The inductance is so large that the output current i_o can be assumed to be continuous and ripple-free. For triggering angle $\alpha = 30°$.

(a) Draw the waveforms of $\boldsymbol{V_{dc}}$ and $\boldsymbol{I_{dc}}$ for highly inductive load.

(b) Prove that the average value of the load voltage is given by:

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\ cos\alpha$$

(c) Determine the average value of the output voltage and current if the phase voltage is $V_{an} = 220$ V (r.m.s), $\alpha = 30°$ and R= 50 ohms.

(d) Find the dc power absorbed by the load.

Solution :

(a) and (b) are found in the lecture notes .

(c)

$$V_{dc} = \frac{3$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{222.93}{50} = 4.45 \text{ A}$$

(d)

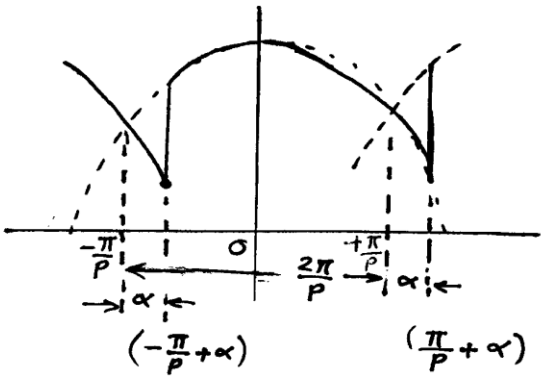
$$p_{dc} = \ V_{dc}\ I_{dc} = 222.93 \times 4.45 = 992 \text{ W}$$

General Solution of P-pulse Half-wave Controlled Rectifier

For p-pulse (or p-phase, i.e 3-phase, 6-phase, 12-phase .. etc) half-wave rectifier circuit, a general formula can be obtained as follows:

It has been found usefull for calculation to express the a.c. voltages on the thyristor side by cosine functions to avoide the mistake in the polarity sign. Hence:

$$v_{an} = V_m \cos(\omega t + 120)$$
$$v_{bn} = V_m \cos \omega t$$
$$v_{cn} = V_m \cos(\omega t - 120)$$



$$V_{dc} = \frac{1}{\frac{2\pi}{p}} \int_{(-\frac{\pi}{p}+\alpha)}^{(\frac{\pi}{p}+\alpha)} V_m \cos\omega t \, d\omega t$$

$$= V_m \left(\frac{p}{2\pi}\right) \left\{ \sin\left(\frac{\pi}{p}+\alpha\right) - \sin\left(-\frac{\pi}{p}+\alpha\right) \right\}$$

$$= V_m \left(\frac{p}{2\pi}\right) \left[\sin\frac{\pi}{p}\cos\alpha + \sin\alpha\cos\frac{\pi}{p} - \sin\left(-\frac{\pi}{p}\right)\cos\alpha - \cos\left(-\frac{\pi}{p}\right)\sin\alpha \right]$$

$$= \frac{p V_m}{2\pi} \left[2 \sin\frac{\pi}{p} \cos\alpha \right]$$

$$V_{dc} = \frac{p}{\pi} V_m \sin\frac{\pi}{p} \cos\alpha$$

8/3ϕ

Or

$$V_{dc} = \frac{V_m}{2\pi / P} \int_{-\frac{\pi}{P}+\alpha}^{\frac{\pi}{P}+\alpha} \cos\ \omega t\ d\omega t = V_m \frac{\sin\frac{\pi}{P}}{\frac{\pi}{P}} \cos\alpha$$

For a 3-phase, half-wave circuit, p= 3, hence

$$V_{dc} = V_m \frac{\sin\frac{\pi}{3}}{\pi/3} \cos\alpha = \frac{V_m \frac{\sqrt{3}}{2}}{\pi/3} \cos\alpha = \frac{3\sqrt{3}\,V_m}{2\pi} \cos\alpha$$

The diode conduction angle is $\frac{2\pi}{3} - \alpha$.

2. Three- phase full – wave fully - controlled rectifier (*p = 6*)

The circuit configuration of the three- phase full – wave controlled rectifier is shown in Fig.7.5. In this circuit, the thyristor which has the most positive voltage at its anode conducts when triggered, and the thyristor with the most negative voltage at its cathode returns the load current, if triggered. The waveforms are shown in Fig. 7.6.

- Commutation of the load current from one thyristor to the next occurs at the firing instant, when the incoming thyristor reverse biases the previously conducting thyristor.
- The output dc voltage waveform is determined by the difference of potentials of the positive and negative rails.

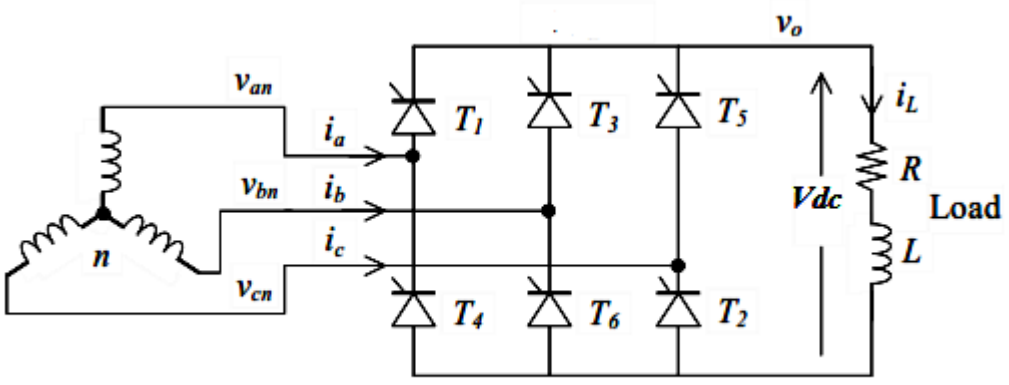


Fig.7.5 Three-phase, fully- controlled bridge converter circuit.

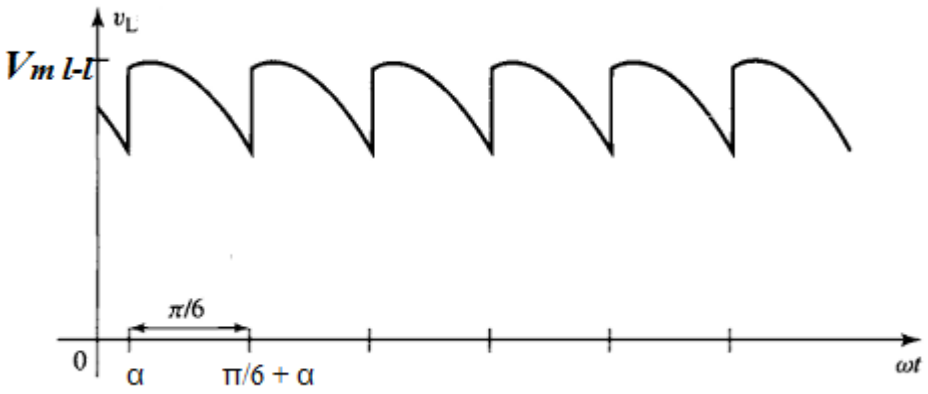


Fig. 7.6 Waveforms

Assuming continuous conduction, the average dc output voltage can be evaluated from the general p-phase formula:

$$V_{dc} = \frac{V_m}{2\pi/P}\int_{-\frac{\pi}{P}+\alpha}^{\frac{\pi}{P}+\alpha}\cos\omega t\, d\omega t = V_m \frac{\sin\frac{\pi}{P}}{\frac{\pi}{P}}\cos\alpha$$

Here p = 6 , $V_{m\ l\text{-}l} = \sqrt{3}V_m$ where $V_{m\ l\text{-}l}$ = maximum line – to –line voltage,

V_m = maximum line –to-neutral voltage. Hence

$$V_{dc} = \sqrt{3}\frac{V_m}{2\pi/6}\int_{-\frac{\pi}{6}+\alpha}^{\frac{\pi}{6}+\alpha}\cos\omega t\, d\omega t = \sqrt{3}V_m \frac{\sin\frac{\pi}{6}}{\frac{\pi}{6}}\cos\alpha$$

$$V_{dc} = \frac{3\sqrt{3}}{\pi}V_m\cos\alpha$$

This converter operates in quadrants 1 and 4, developing both positive and negative polarity dc output voltage. For firing angles, $0^o \le \alpha \le 90^o$ the converter operates in quadrant 1 (giving positive output power, i.e., rectifier operation) and for $90^o \le \alpha \le 180^o$, the operation is in quadrant 4 (giving negative output power, i.e., inverter operation). Operation in quadrant 4 is of course possible only when the load includes an active dc source, able to source power into the ac supply circuit. See Fig.7.7.

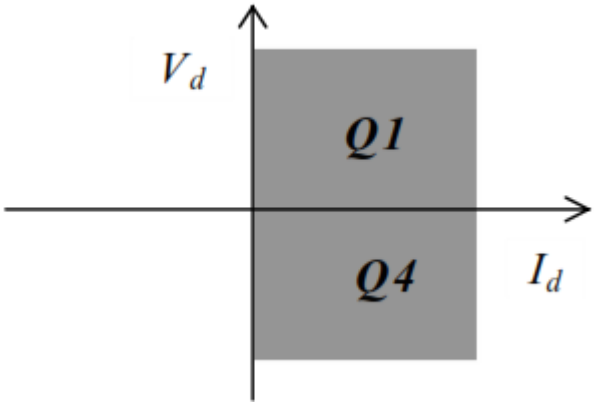


Fig. 7.7 Operating quadrants of the 3-phase Fully-C controlled converter

3. Three- phase full – wave , Half -controlled rectifier

This converter is shown in Fig.7.8. It consists of three thyristors and three diodes with freewheeling diode across the load. It gives positive voltage and positive current only (not regenerative converter) i.e, it operates in the first quadrant only (Fig.7.9).

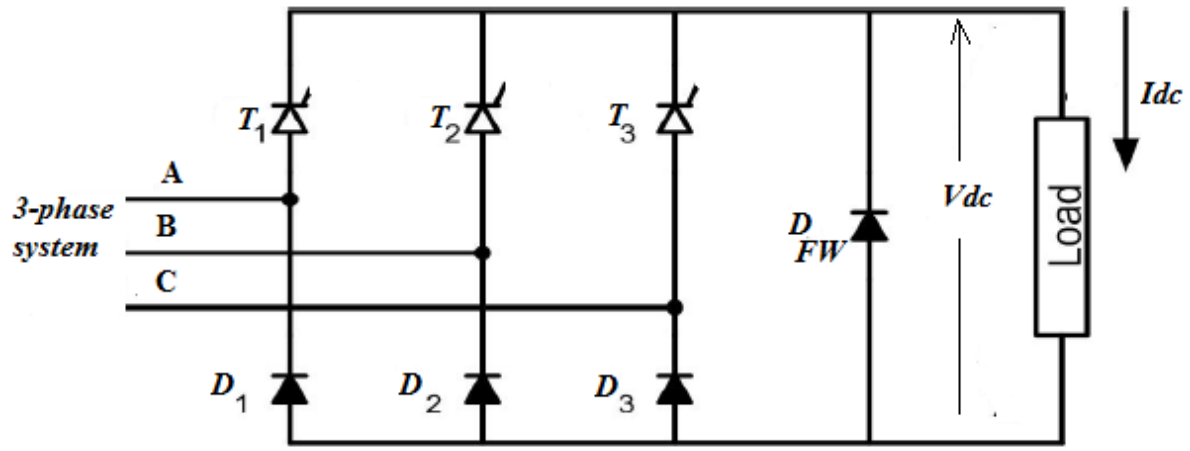


Fig.7.8

The output voltage is given by:

$$V_{dc} = \frac{3\sqrt{3}}{2\pi} V_m (1 + \cos\alpha)$$

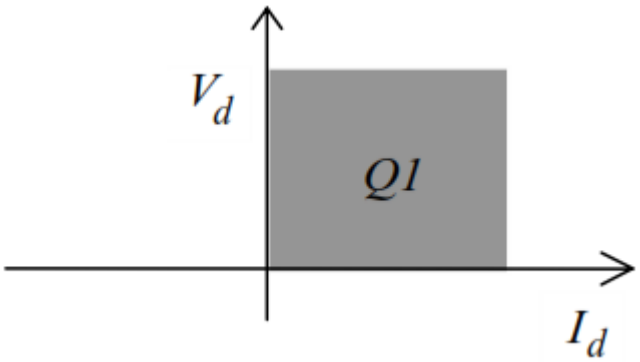


Fig.7.9

Example 1: A 3-phase Full-wave Fully controlled rectifier supply a highly inductive load with $R = 10\,\Omega$. The supply is a 3-phase Y-connected with 208 r.m.s. Calculate:

(a) The load current when the firing angle $\alpha = 40^\circ$.

(b) The power drawn from the supply.

(c) If the current value kept at (a) and α changed to 135°, calculate the power returned to the supply.

Solution

(a) For $\alpha = 40^\circ$

$$V_m = \frac{\sqrt{2} \times 208}{\sqrt{3}} = 169.83 \text{ V} \qquad \text{(per phase)}$$

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{\pi} \cos\alpha = \frac{3 \times \sqrt{3} \times 169.83}{\pi} \cos 40^\circ = 215.18 \text{ V}$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{215.18}{10} = 21.518 \text{ A}.$$

(b) The power drawn from the source = the power dissipated at the resistance of the load

$$P_s = P_{load} = V_{dc}\, I_{dc} = 215.18 \times 21.518 = 4630.25 \text{ W}$$

(c) For $\alpha = 135^\circ$, $i_o = I_{dc} = 21.518 \text{ A}$

$$V'_{dc} = \frac{3\sqrt{3}\,V_m}{\pi} \cos 135^\circ = \frac{3\sqrt{3} \times 169.83}{\pi} \cos(135)$$

$$= -198.625.$$

$\therefore$ Power return to the source:

$$P_s = V'_{dc}\, I_{dc} = -198.625 \times 21.518$$

$$= -3945.189 \text{ W}.$$

Example 2 : If the converter in example .7 is replaced by Full-wave half-controlled converter, Calculate :

(a) V_{dc} when $\alpha = 45^\circ$
(b) V_{dc} when $\alpha = 75^\circ$
(c) V_{dc} when $\alpha = 135^\circ$
(d) Maximum value of V_{dc} .
(e) The value of α to obtain $I_{dc} = 6A$.

Solution :

(a) $$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}(1+\cos\alpha)$$

For $\alpha = 45^\circ$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 45^\circ) = 239.76\ V.$$

(b) For $\alpha = 75^\circ$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 75^\circ) = 176.8\ V.$$

(c) For $\alpha = 135^\circ$

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 135^\circ) = 41.36V.$$

(d) Max. voltage output is when $\alpha = 0^\circ$:

$$V_{dc} = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos 0^\circ) = 280.8\ V.$$

(e) $I_{dc} = \frac{V_{dc}}{R} = 6A$ or $V_{dc} = 6R = 6 \times 10 = 60V.$

$$60 = \frac{3\sqrt{3} \times 169.83}{2\pi}(1+\cos\alpha)$$

From which $\alpha = \underline{52.9^\circ}$.

8. DC Drives

This senior part of electrical engineering introduces the principles of combining Electrical Machines with Power Electronics Converters. This combination is commonly known as Electrical Drives.However, Electrical Drives can be classified as DC Drives and AC Drives .In this lecture, we shall consider DC drives only.

1. Power Semiconductor Drive and its Elements

The basic arrangement of an electrical drive is shown in Fig.8.1 .The block digram of this DC drive is shown in Fig.8.2.

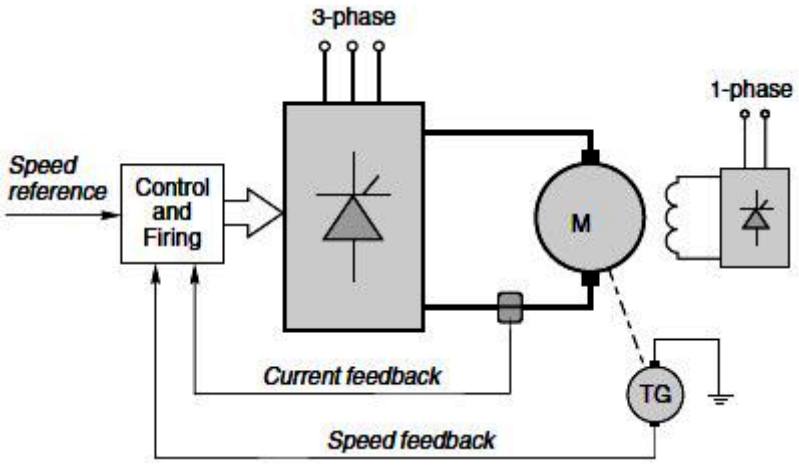


Fig.8.1 Schematic diagram of speed-controlled d.c. motor drive.

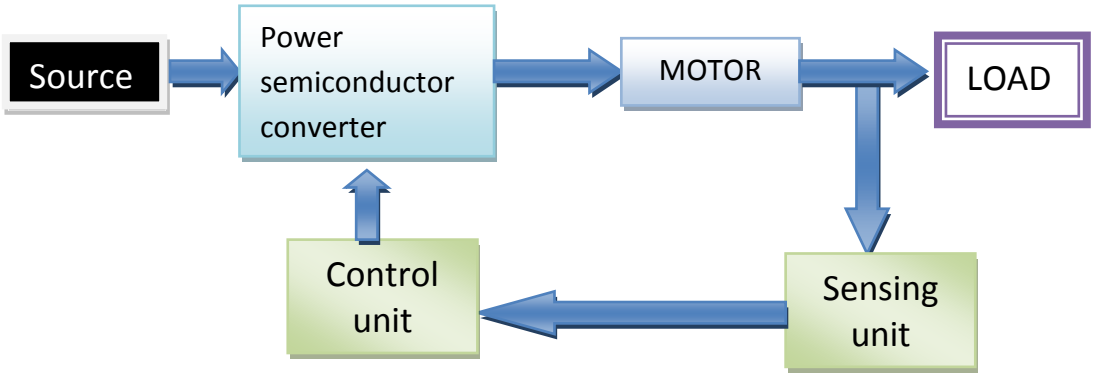


Fig.8.2 Block diagram of the d.c. drive.

For motors up to a few kilowatts the armature circuit of the motor can be supplied from either single-phase or three-phase mains, but for larger motors three-phase is always used. A separate thyristor or diode rectifier is used to supply the field of the motor: the power is much less than the armature power, so the supply is often single-phase, as shown in Figure 8.1. An example of actual DC drive circuit with the armature of the motor supplied from a three-phase full-wave fully controlled rectifier is shown in fig.8.3.

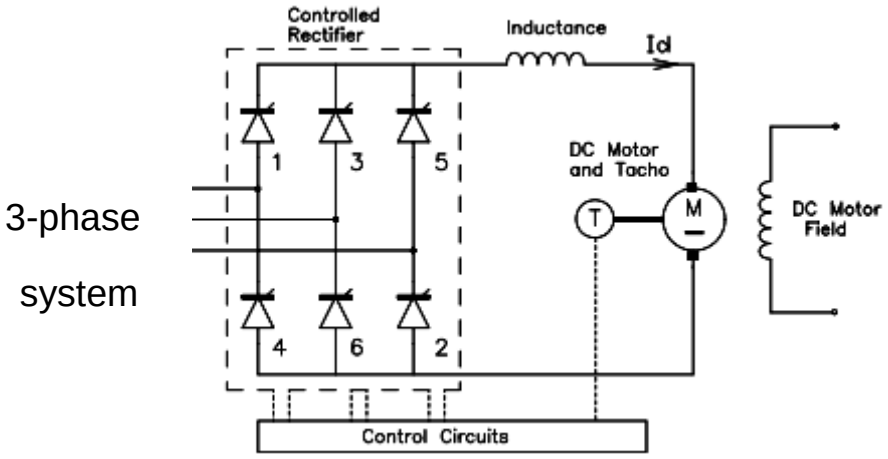
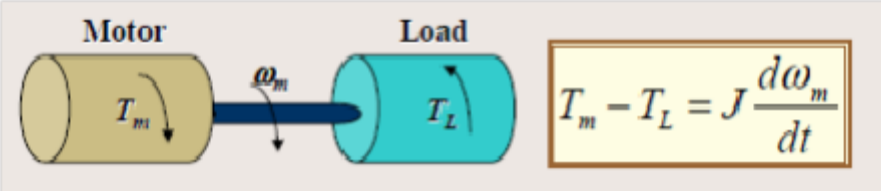


Fig.8.3 Example of actual DC drive circuit

2. Dynamics of the Load System



J = *p*olar moment of inertia of motor-load system referred to the motor shaft, Kg.m2

ω_m = instantaneous angular velocity of the motor shaft, rad / sec

$T_m = T_d$ = developed torque of the motor, N.m

T_L=load (resisting) torque, referred to the motor shaft, N.m

Note:

$$\text{At steady-state:} \quad \frac{d\omega_m}{dt} = 0 \Rightarrow T_m = T_L$$

$$J\frac{d\omega_m}{dt} = \text{Dynamic torque}$$

$T_m > T_L$: acceleration

$T_m < T_L$: deceleration

In this course notes we shall consider the motor – drive system operation in the steady – state only.

3. Review of DC Motor Theory

DC motor construction:

Dc motor is mainly constructed from two main parts, the stator and the rotor as shown in Fig.8. 4.

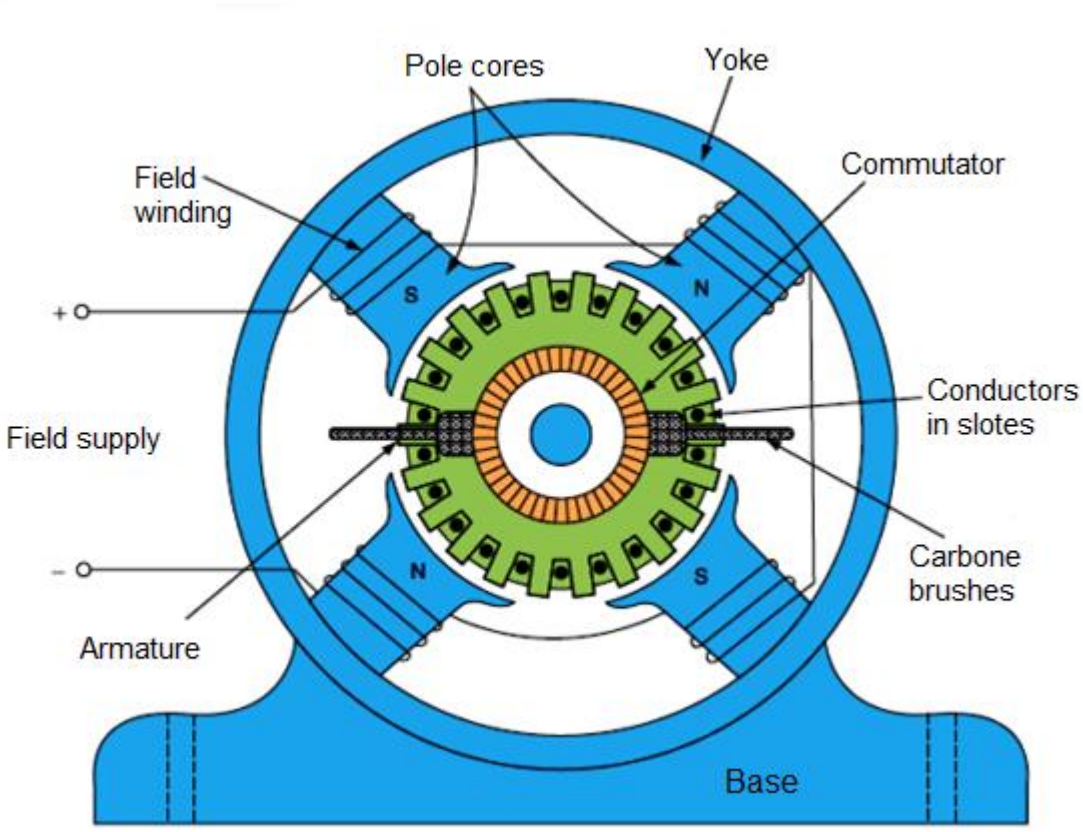


Fig.8.4 DC machine construction

Types of DC motors: There are several types of dc motors:

1. Separately Excited motor
2. Shunt DC Motor
3. Series motor
4. Compound motor

The circuit connection diagrams of these four types of dc motors are shown in Fig.8.5.

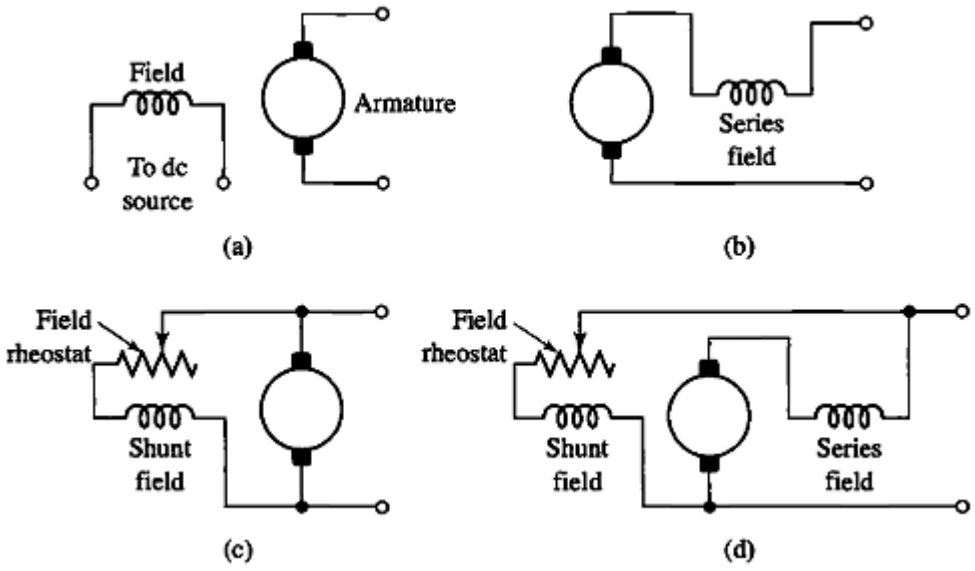


Fig.8.5.Connection of the field circuit and armature circuit for (a) separately exited motor,(b) series motor (c) shunt motor and (d) compound motor.

In this lecture we shall consider mainly the separately excited and the shunt DC motors.

3.1 Separately Excited and Shunt DC Motors

A separately excited dc motor is a motor whose field circuit is supplied from a separate constant-voltage power supply, while a shunt dc motor is a motor whose field circuit gets its power directly across the armature terminals of the motor.

When the supply voltage to a motor is assumed constant, there is no practical difference in behaviour between these two machines. Unless otherwise specified, whenever the behaviour of a shunt motor is described, the separately excited motor is included too.

The equivalent circuits of these two DC motors are shown in Fig.8.6

The KVL equation for the armature circuit is:

$$V_T = E_A + I_A R_A$$

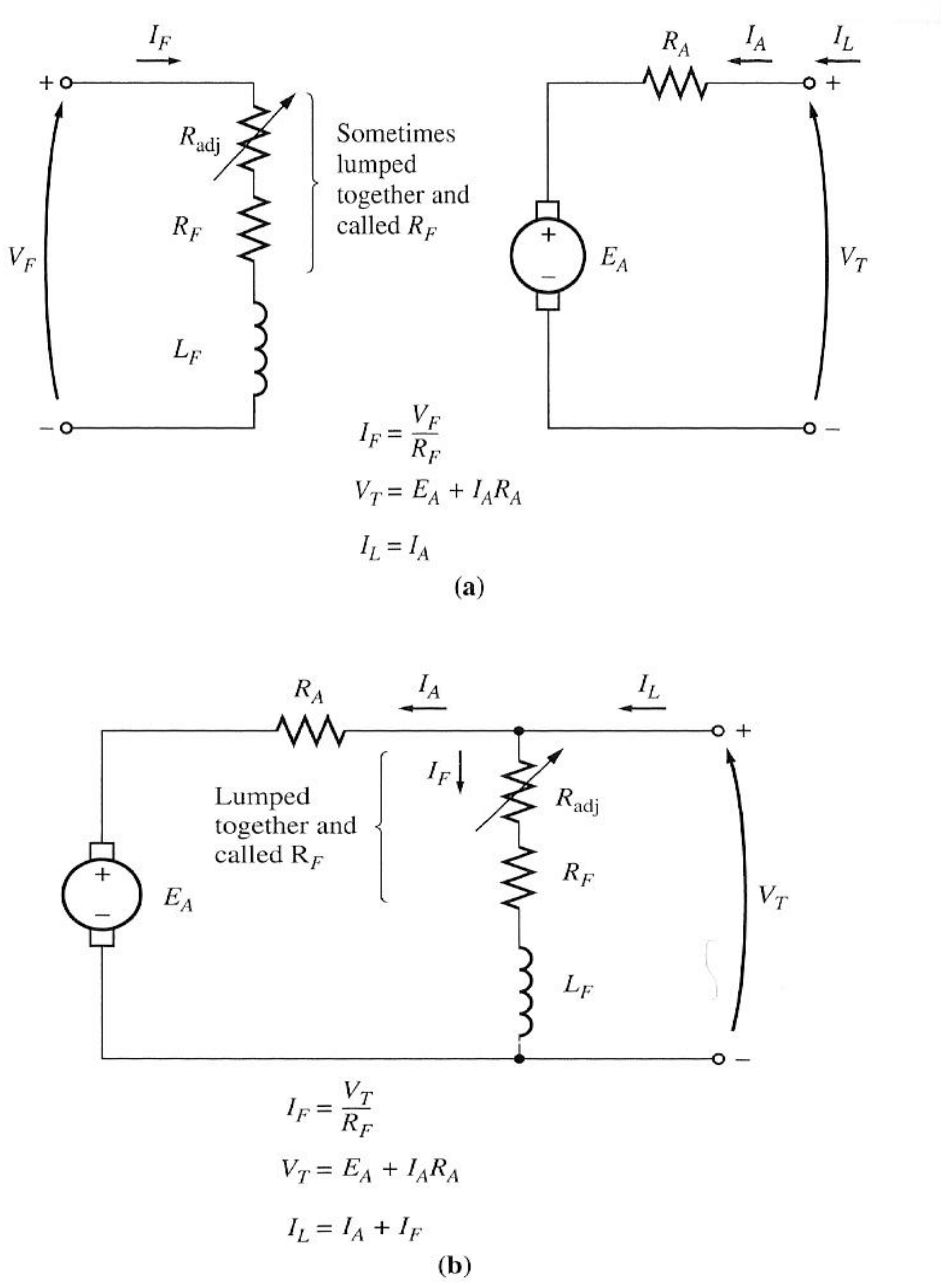


Fig.8.6 The equivalent circuit of (a) separately excited dc motor and (b) shunt dc motor.

The internal generated voltage (back emf) is given by:

$$E_A = K_e \phi n$$

Where n= speed of the motor in revolution per minute (rpm),

Φ = flux per pole in Weber (Wb),

K_e = Machine constant = p.Z / 60a

p = number of poles, Z=total number of conductors , a = number of parallel paths (a= p for lap winding , a= 2 for wave winding).

and the developed motor torque is

$$T_d = K_T I_A \varphi$$

K_T = Torque constant = 9.55 K_e , I_A = armature current (A).

The speed equqation and the terminal characteristics of a DC Motor

A terminal characteristic of a dc machine is a plot of its output torque versus speed.

The output characteristic of a separately excited and shunt dc motors are approximately the same and can be derived from the induced voltage and torque equations of the motor plus the KVL as follows:

KVL → $V_T = E_A + I_A R_A$

The induced voltage $E_A = K_e \phi n$

$$V_T = K_e \varphi n + I_A R_A$$

Since $T_d = K_T I_A \varphi$

current I_A can be expressed as: $I_A = \frac{T_d}{\emptyset K_T}$

Combining the V_T and I_A equations:

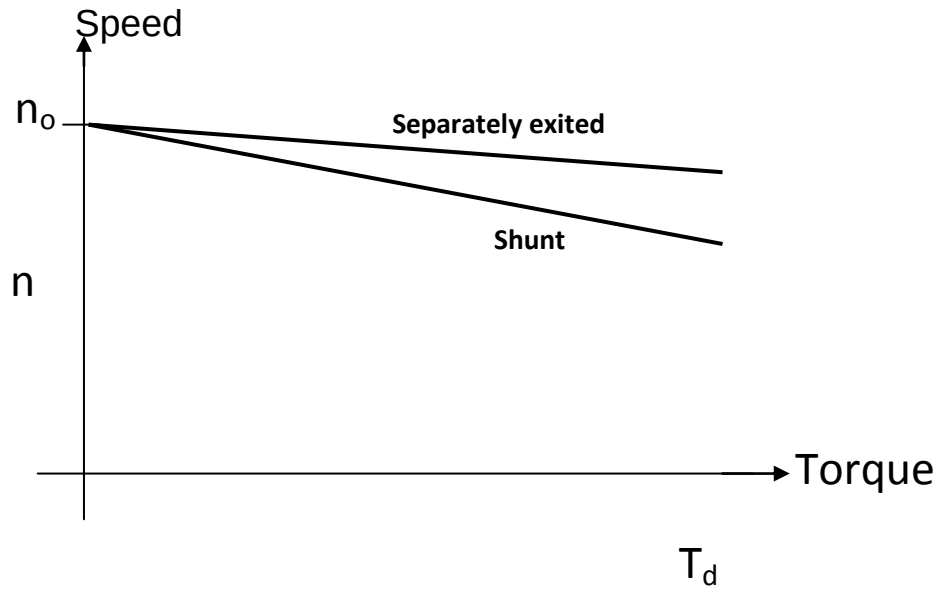
$$V_T = K_e \emptyset n + \frac{T_d}{K_T \emptyset} R_A$$

Finally, solving for the motor's speed:

$$n = \frac{V_T}{K_e \emptyset} - \frac{R_A}{K_T K_e \emptyset^2} T_d$$

Where n= speed in rpm.

This equation is called the dc motor speed equation and is just a straight line with a negative slope. The resulting torque-speed characteristic of a shunt dc motor is shown in fig.8.7:



Fig, 8.7 Speed - torque characteristic of a shunt or separately excited dc motor.

Where : n_o = no load speed (i.e. when T_d =0) or

$$n_o = \frac{V_T}{K_e \emptyset}$$

Speed Control of Shunt and sepatately excited DC Motors

Two common methods

I - Adjusting the field resistance R_F (and thus the field flux ϕ)

ii Adjusting the terminal voltage applied to the armature.

Less common method:

Iii-Inserting a resistor in series with the armature circuit.

In this lecture we shall consider mainly the DC motors spead control using armature voltage control by AC-to-DC converters .These single-

phase and three-phase converters (drives) are shown in Figs 8.8 and 8.9.respectively.

Types of DC Drives
1.Single-phase DC Drives

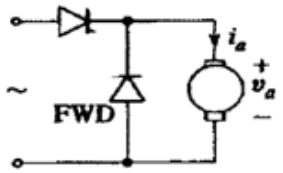
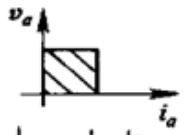
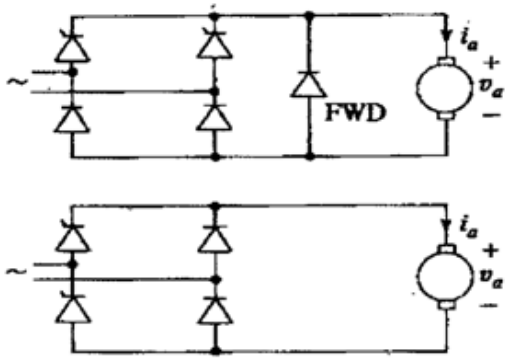
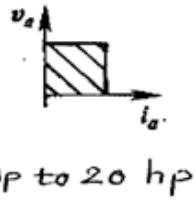
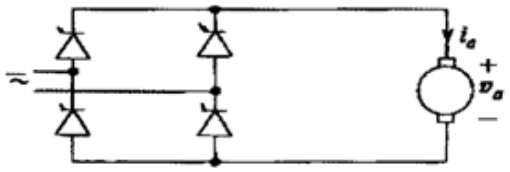
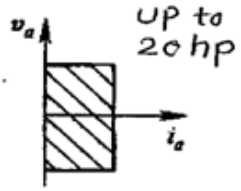
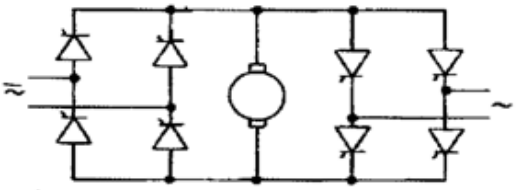
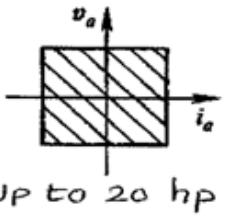
Type	Circuit	Operation
half-wave 1 pulse		
semi-converter 2 pulse		
full converter 2 pulse		
double converter 2 pulse		

Fig.8.8

2. Three-phase DC Drives

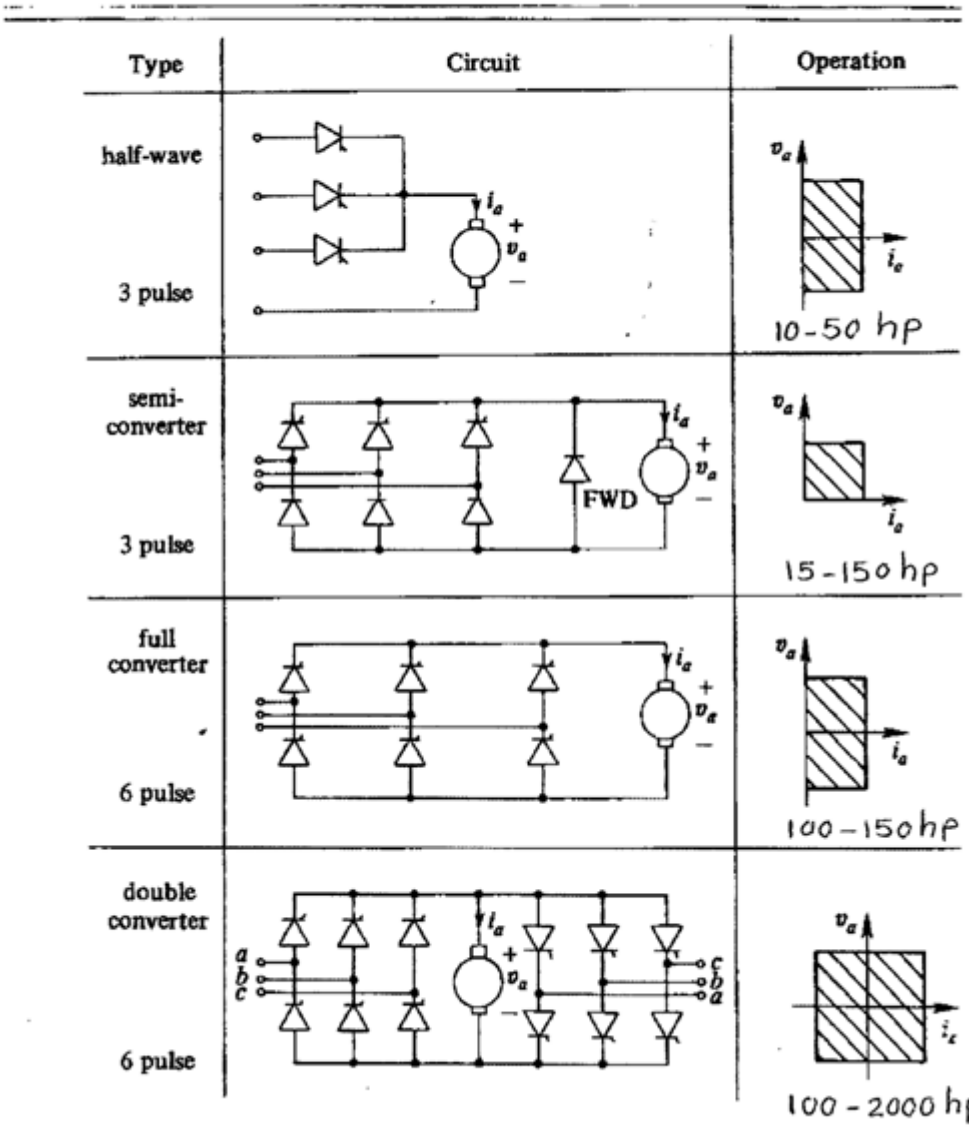


Fig.8.9

Example 1: A separately excited DC motor has the following parameters:

Ra = 0.3 Ω , Ke = 0.52 V/rpm.Wb , Φ (flux per pole) = 150 mWb.

The motor speed is controlled by a full wave bridge rectifier. The firing angle α is set at 60°, and the average speed is 1250 rpm. The applied a.c. voltage to the bridge is 230 V at 50 Hz . Assuming the motor current is continuous; calculate the armature current drawn by the motor and the steady-state torque for the cases of:

(a) Fully- controlled bridge

(b) Half-controlled (semiconverter) bridge.

Solution :

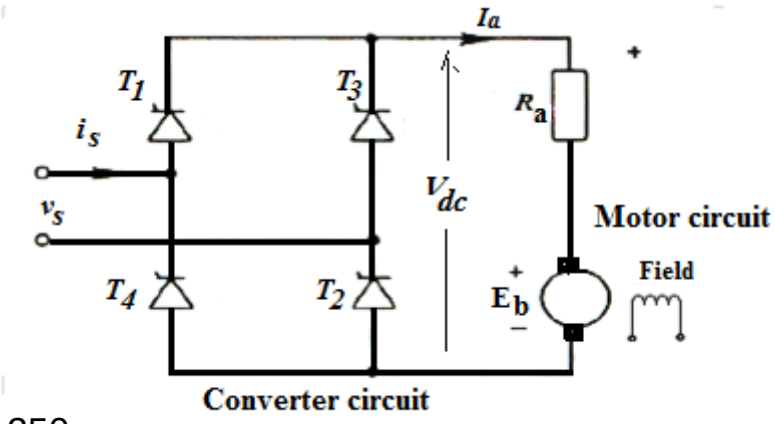
(a) For fully- controlled bridge with continuous current operating mode:

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha$$

V_m=√2 x 230 =325.2V

$$V_{dc} = \frac{2\,x325.2}{\pi}\cos 60$$

$$V_{dc} = 104.4V$$



$E_b = K_e \varphi n = 0.52x150x10^{-3}x1250$

= 97.5V

$V_T = V_{dc} = E_a + I_a R_a$

Or

$$I_a = \frac{V_{dc} - E_b}{R_a} = \frac{104.4 - 97.5}{0.3} = 23\ A$$

Since $T_d = K_T I_a \varphi$

K_T = Torque constant = 9.55 K_e = 9.55 x 0.52 = 4.96

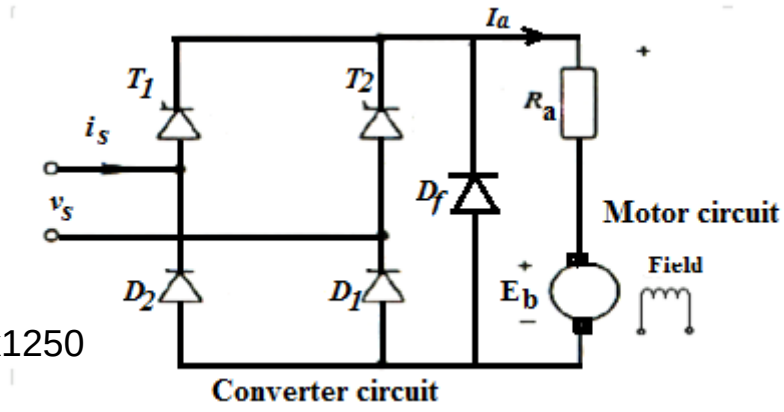
T_d = 4.96 x 23 x 150 x 10^{-3} =17.1 N

(b) For half-controlled (semiconverter) bridge.

$$V_{dc} = \frac{V_m}{\pi}(1 + cos\alpha)$$

$$V_{dc} = \frac{325.2}{\pi}(1 + cos60)$$

$$V_{dc} = 155.3\ V$$



$E_b = K_e \varphi n$ =0.52x150x10^{-3}x1250

= 97.5V

$V_T = V_{dc} = E_a + I_a R_a$

$$I_a = \frac{V_{dc} - E_b}{R_a} = \frac{155.3 - 97.5}{0.3} = 192.8\ A$$

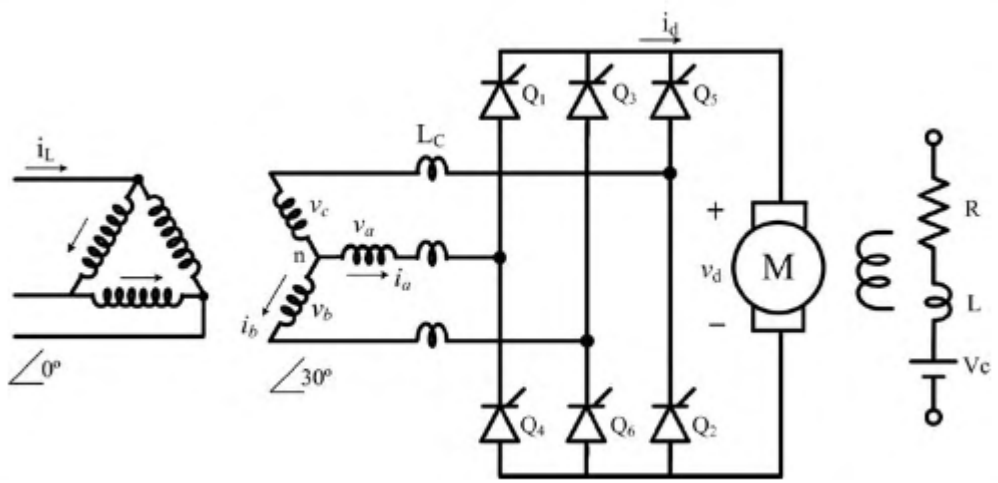
T_d = 4.96 x 192.8 x150 x 10^{-3} = 146.4 N

Example 2 : A three-phase full converter , shown in Fig.8.10 , is used to control the speed of a separately excited d.c. motor rated at 100 kW, 600V, 2000 rpm. The converter is connected to a 3 – phase 400 V, 50 Hz supply. The armature resistance is Ra = 0.051 ohm and the armature circuit inductance is La = 10 mH. The motor voltage constant is Ke Φ = 0.25 V/rpm. The rated armature current is 100A and the no load current is 10A.

With the converter operates as a rectifier, and assuming that the motor current is continuous and ripple-free, determine:

(a) The no load speed when the firing angles: α= 0º and α= 60º.
(b) The firing angle to obtain the rated speed of 2000 rpm at rated motor current.

Fig.8.10



Solution:

(a) No load condition

$$V_m = \frac{400}{\sqrt{3}} x\sqrt{2} = 325.22\ V$$

The converter output voltage=Vdc = armature voltage V_T

$$V_{dc} = V_T = \frac{3\sqrt{3}}{\pi} V_m \cos\alpha$$

$$V_T = \frac{3\sqrt{3}}{\pi} x325.22 \cos\alpha$$

$$V_T = 538 \cos\alpha$$

For α= 0º $V_T = 538$ V

$E_A = V_T - I_A R_A = 538 – 10x0.051 = 537.5$ V

No load speed :

since $E_A = K_e \varphi n$

Hence

$$n_o = \frac{E_A}{K_e \emptyset} = \frac{537.5}{0.25} = 2145\ rpm$$

For α= 60º

$V_T = 538 \cos 60º = 269$ V

$E_A = V_T - I_A R_A = 269 – 10x0.051 = 268.4$ V

$$n_o = \frac{E_A}{K_e \emptyset} = \frac{268.41}{0.25} = 1073.64\ rpm$$

(b) At full load condition

$E_A = K_e \varphi n = 0.25 \times 2000 = 500$V

$V_T = E_A + I_A R_A = 500 + 100 \times 0.051$

$= 505$ V

$505 = 538 \cos\alpha$

Hence α =20.14º

Dual converters

The dual converter consists of two AC-to-DC converters connected in anti-parallel as shown in Fig, 8.11.If converter 1 operates, and it gives a positive output dc voltage (+ V_{dc}) .However if converter2 operates it gives negative dc voltage -V_{dc}. It is important to be noted that, the two converters must not operate simultaneously to avoid short - circuiting of the two converters.

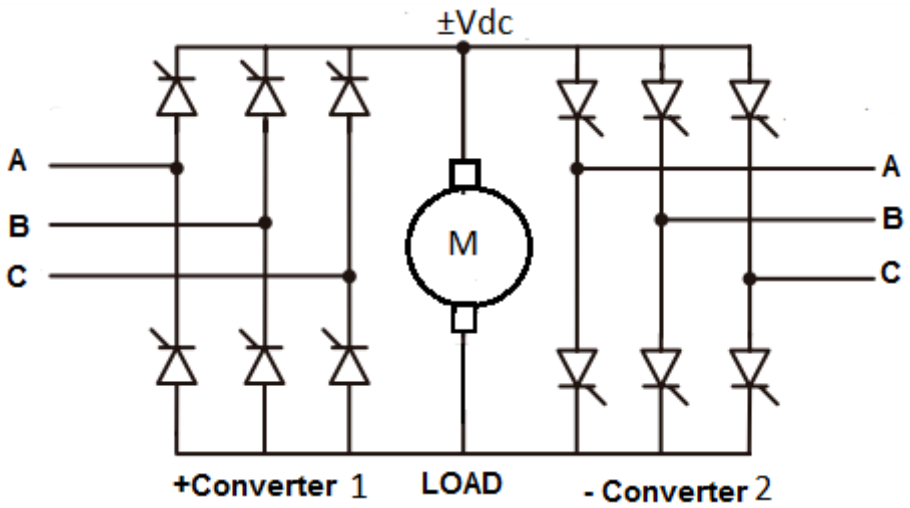


Fig.8.11

The dual converter provides the facility of operation in four quadrants (Fig.8.12). If the load is a dc motor, the motor can runs in four modes of operation as indicated in Fig.8.13.

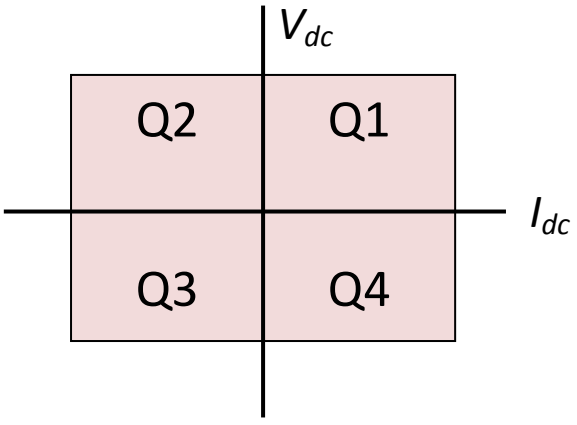


Fig.8.12

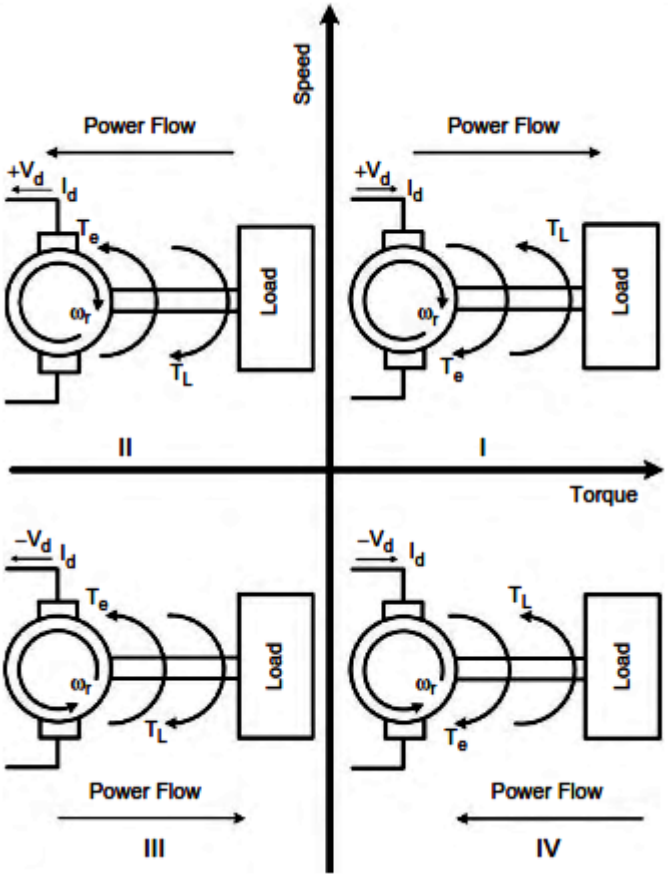

Speed
Power Flow
Power Flow
+Vd
Id
Te
ωr
Load
TL
TL
Te
II
I
Torque
–Vd
Id
Te
TL
Power Flow
Power Flow
III
IV

Fig.8.13

9. DC-DC Converters (DC –Choppers)

A dc-to-dc converter, also known as dc chopper, is a static device which is used to obtain a variable dc voltage from a constant dc voltage source. Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc….. They are also used as dc voltage regulators.

Choppers are of two types: (1) Step-down choppers, and (2) Step-up choppers. In step-down choppers, the output voltage will be less than the input voltage, whereas in step-up choppers output voltage will be more than the input voltage.

9.1 PRINCIPLE OF STEP-DOWN CHOPPER

Figure 9.1 shows a step-down chopper with resistive load. The thyristor in the circuit acts as a switch. When thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage waveform is as shown in Fig. 9.2.

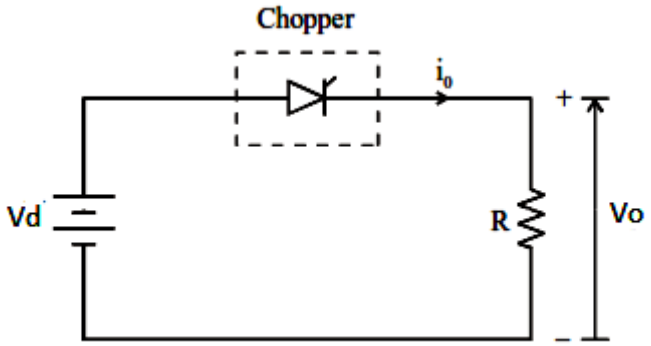


Fig.9.1 Chopper circuit.

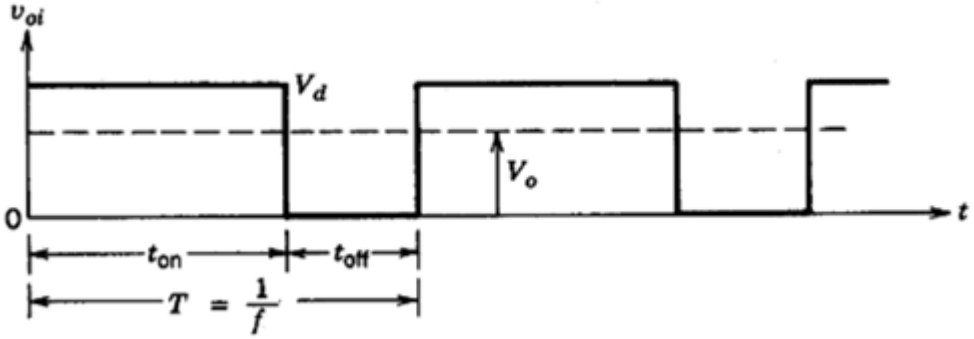


Fig.9.2 Chopper output voltage waveform, R- load.

Methods of Control

The output dc voltage can be varied by the following methods.

- ❖ Pulse width modulation control or constant frequency operation.
- ❖ Variable frequency control.

Pulse Width Modulation

- t_{ON} is varied keeping chopping frequency '*f*' & chopping period 'T' constant.
- Output voltage is varied by varying the ON time t_{ON}

9.2 ANALYSIS OF A STEP-DOWN CHOPPER WITH R-LOAD

Referring to Fig.9.2, the average output voltage v_o can be found as

Let T= control period = $t_{on} + t_{off}$

$$V_o = V_{av} = \frac{1}{T}\int_0^{t_{on}} V_d \; dt$$

$$V_o = V_d \frac{t_{on}}{T} = V_d(\gamma)$$

$$where\ , \gamma = \frac{t_{on}}{T} = Duty\ cycle$$

- Maximum value of $\gamma = 1$ when $t_{on} = T$ ($t_{off} = 0$)
- Minimum value of $\gamma = 0$ when $t_{on} = 0$ ($t_{off} = 0$)

The output voltage is stepped down by the factor γ $(0 \le V_o \le V_d)$ .Therefore this form of chopper is a step down chopper.

The average value of the output current in case of resistive load:

$$I_{av} = \frac{V_{av}}{R} = \frac{V_o}{R} = \frac{\gamma V_d}{R}$$

The d.c. power is

$$P_{dc} = V_o\, I_{av} = I_{av}^2 R = (\frac{\gamma V_d}{R})^2\, R$$

The R.M.S. value of the output voltage

$$V_{o,rms} = \sqrt{\frac{1}{T}\int_0^{t_{on}} v_o^2\, dt} = \sqrt{\gamma}\, V_d$$

The rms value of the output current

$$I_{o,rms} = \frac{V_{o,rms}}{R} = \frac{\sqrt{\gamma}\, V_d}{R}$$

The a.c. power output is

$$P_{ac} = V_{o,rms}\, I_{o,rms} = I_{o,rms}^2 R$$

f= chopping frequency = $(\frac{1}{chopping \quad period\,(T)})$ = 1/ T

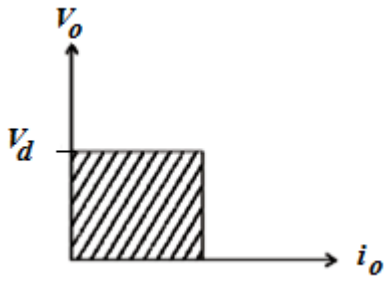
The ripple factor, *RF*

It is a measure of the ripple content.

$$RF = \sqrt{(\frac{Vo_{rms}}{Vo})^2 - 1} = \sqrt{(\frac{\gamma V_d^{\,2}}{\gamma^2 V_d^2}) - 1} = \sqrt{\frac{1}{\gamma} - 1} = \sqrt{\frac{1-\gamma}{\gamma}}$$

Note1: In this type of chopper both the voltage and current are always positive, hence this chopper is called a single-quadrant Buck converter or class – A chopper.

Fig.9.3 Single – quadrant operation



Note2: The chopper switch can also be implemented by using a power BJT, power MOSFET, GTO, and IGBT transistor. The practical devices have a finite

voltage drop ranging from 0.5V to 2V, and for the sake of simplicity, the voltage drop of their power semi-conductor devices are neglected.

Example 1: A transistor dc chopper circuit (Buck converter) is supplied with power form an ideal battery of 100 V. The load voltage waveform consists of rectangular pulses of duration 1 ms in an overall cycle time of 2.5 ms. Calculate, for resistive load of 10 Ω.

(a) The duty cycle γ.

(b) The average value of the output voltage V_o.

(c) The rms value of the output voltage V_{orms}.

(d) The ripple factor *RF*.

(e) The output d.c. power.

Solution:

(a) t_{on} = 1ms , T=2.5 ms

$$\gamma = \frac{t_{on}}{T} = \frac{1\ ms}{2.5\ ms} = 0.4$$

(b) $V_{av} = V_o = \gamma\ V_d = 0.4 \times 100 = 40$ V.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.4} \times 100 = 63.2$ V.

(d) $RF = \sqrt{\frac{1-\gamma}{\gamma}} = \sqrt{\frac{1-0.4}{0.4}} = 1.225$

(e)

$$I_a = \frac{V_o}{R} = \frac{40}{10} = 4A$$

$P_{av} = I_a\ V_o = 4 \times 40 = 160$ W

9.3 STEP-DOWN CHOPPER WITH R-L LOAD

Consider a class-A chopper circuit with R-L load as shown in Fig.9.4.This is a step down chopper with one quadrant operation.

If we use the simplified linear analysis by considering that $T << \tau$, where ($T = t_{on} + t_{off}$) . In this case the current is continuous as shown in Fjg.9.5.

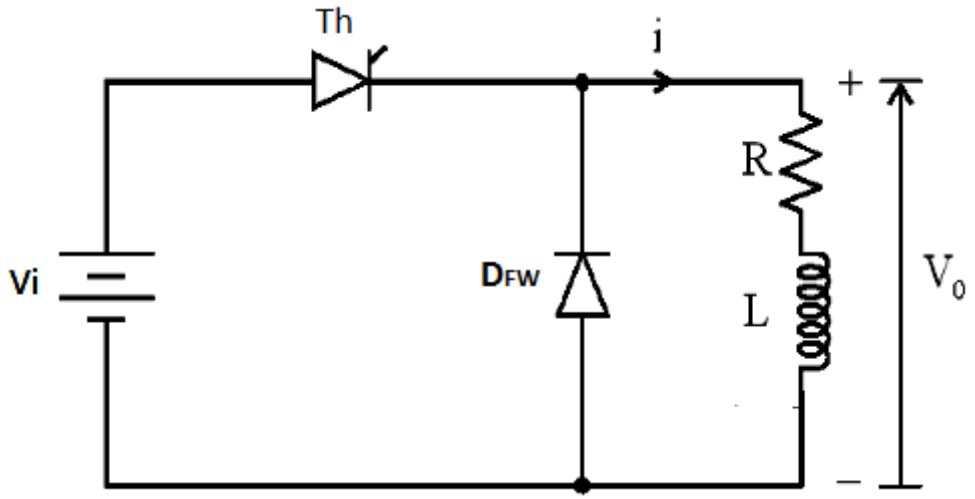


Fig.9.4

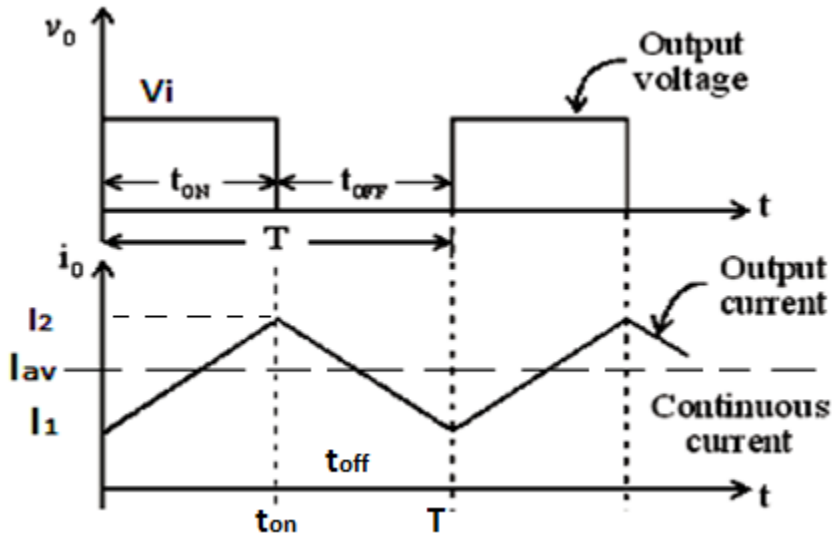


Fig.9.5

Referring to Fig.9.5:

- The current variation is almost linear and the current waveform becomes triangular.
- During the ON period , the equation govern the circuit is

$$V_d = Ri + L\frac{di}{dt}$$

Since $\frac{di}{dt}$ = constant, hence during ON period:

$$\frac{di}{dt} = \frac{I_2 - I_1}{t_{on}} = \frac{\Delta I}{t_{on}}$$

Where ΔI is the peak – to –peak of the load current .Thus the equation of the current is given by:

$$i_1 = I_1 + \frac{\Delta \mathrm{I}}{\gamma T} t \qquad\qquad 0 \le t \le t_{on}$$

Where $\gamma = \frac{t_{on}}{T}$

During the off period:

$$\frac{di}{dt} = \frac{I_1 - I_2}{t_{off}} = -\frac{\Delta I}{t_{off}} = -\frac{\Delta \mathrm{I}}{T - t_{on}} = -\frac{\Delta \mathrm{I}}{T - \gamma T} = -\frac{\Delta \mathrm{I}}{(1-\gamma)T}$$

Hence, during the off the equation of the current is

$$i_2 = I_2 - \frac{\Delta \mathrm{I}}{(1-\gamma)T}\,(t - t_{on}) \qquad\qquad t_{on} \le t \le T$$

The average value of the output current is

$$I_{av} = \frac{1}{T}\left[\frac{1}{2}\, t_{on}(I_2 - I_1) + \frac{1}{2}\, t_{off}(I_2 - I_1) + I_1 T\right]$$

$$I_{av} = \tfrac{1}{2}(I_2 + I_1)$$

Example 2: An 80 V battery supplies RL load through a DC chopper. The load has a freewheeling diode across it is composed of 0.4 H in series with 5Ω resistor. Load current, due to improper selection of frequency of chopping, varies widely between 9A and 10.2.

(a) Find the average load voltage, current and the duty cycle of the chopper.

(b) What is the operating frequency *f* ?

(c) Find the ripple current to maximum current ratio.

Solution:

(a) The average load voltage and current are:

$V_{av} = V_o = \gamma V_d$

$$I_{av} = \frac{1}{2}(I_2 + I_1) = \frac{9 + 10.2}{2} = 9.6A$$

$$I_{av} = \frac{V_{av}}{R} = \frac{\gamma V_d}{R} \qquad or \qquad \gamma = \frac{I_{av}\ R}{V_i} = \frac{9.6\ x\ 5}{80} = 0.6$$

V_{av} = 0.6 x 80 = 48 V.

(b) To find the operating (chopping) frequency:

During the ON period,

$$V_d = Ri + L\frac{di}{dt} \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Assuming $\frac{di}{dt} \cong constant$

$$\frac{di}{dt} \cong \frac{\Delta I}{t_{on}} = \frac{10.2 - 9}{\gamma T} = \frac{1.2}{\gamma T}$$

From eq.(1)

$$L\frac{di}{dt} \cong V_d - I_{av}\ R = 80 - 5 \times 9.6 = 32V$$

or

$$\frac{di}{dt} = \frac{32}{L} = \frac{32}{0.4} = 80\ A.s$$

but

$$\frac{di}{dt} = \frac{1.2}{\gamma T} = 80 = \frac{1.2}{0.6\,T}$$

$$\therefore T = \frac{1.2}{0.6x80} = 25\ ms$$

Hence

$$f = \frac{1}{T} = \frac{1}{25 \times 10^{-3}} = 40\ Hz$$

The maximum current I_m occurs at $\gamma = 1$,

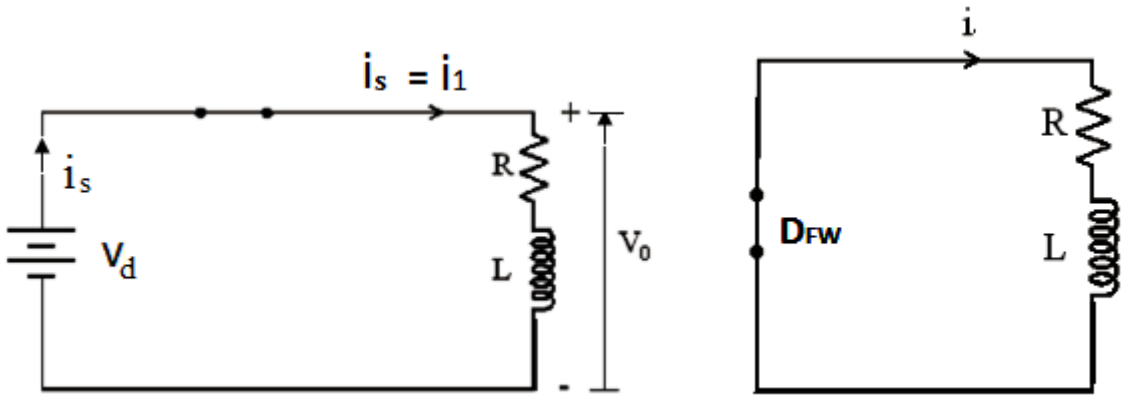
$$\therefore\ I_m = \frac{\gamma V_d}{R} = \frac{1 \times 80}{5} = 16A$$

Ripple current $I_r = \Delta I = 10.2 - 9 = 1.2$ A

$$\therefore\ \frac{I_r}{I_m} = \frac{1.2}{16} = 0.075 \quad or\ \ 7.5\%\ .$$

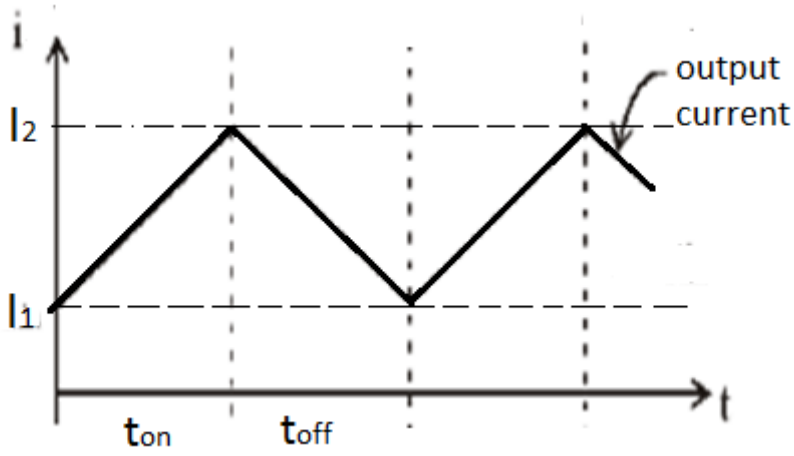
Input Current I_s (I_{in})

For the class-A chopper had shown in Fig.9.4, the On-state and OFF-state equivalent circuits are as depicted in Fig.9.6. When the thyristor is closed (during the ON period), the load current "i" rises from I_1 to I_2 and falls from I_2 to I_1 during the off period as shown in Fig.9.7(a).The input current i_s flows during the ON period only as shown in Fig .9.7(b).

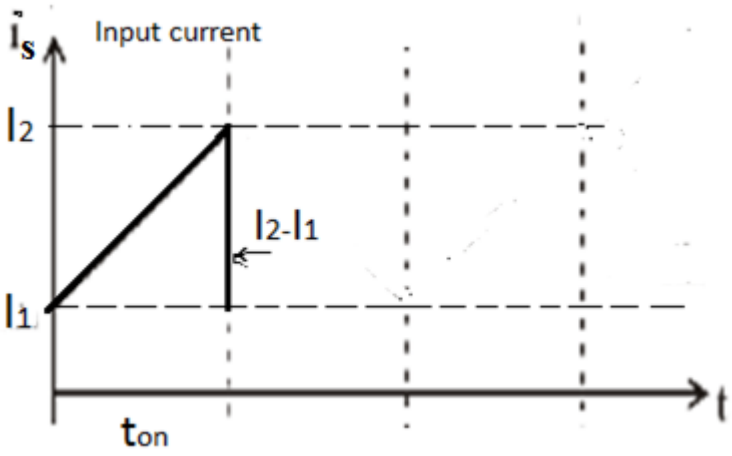


ON- State Equivalent CCT OFF – State Equivalent CCT

Fig.9.6



(a)



(b)

Fig.9.7

The equation of the input current is

$$i_s = i_1 = I_1 + \frac{\Delta I}{\gamma T} t \qquad 0 \le t \le t_{on}$$

$$i_s = 0 \qquad t_{on} \le t \le T$$

The average value of the current drawn from the supply is simply found by,

$$I_{s(av)} = \frac{1}{T}\left[\frac{1}{2}\, t_{on}(I_2 - I_1)\right] + \frac{t_{on} I_1}{T}$$

$$I_{s(av)} = \frac{1}{T}\left[\frac{1}{2}\, t_{on}(I_2 + I_1)\right] = \frac{t_{on}}{2T}\,(I_2 + I_1) = \gamma I_{av}$$

Minimum and Maximum Load Currents

The minimum current I_1 and maximum current I_2 can be found from the following two equations:

$$I_1 = I_{min} = \frac{V_o}{R} - \frac{t_{off}}{2L} V_o$$

$$I_2 = I_{max} = \frac{V_o}{R} + \frac{t_{off}}{2L} V_o$$

Where $\boldsymbol{V_o = V_{av}}$

Note: The proof of these two equations is not needed

Example 3: A DC Buck converter operates at frequency of 1 kHz from 100V DC source supplying a 10 Ω resistive load. The inductive component of the load is 50mH.For output average voltage of 50V volts, find:

(a) The duty cycle

(b) t_{on}

(c) The rms value of the output current

(d) The average value of the output current

(e) I_{max} and I_{min}

(f) The input power

(g) The peak-to-peak ripple current.

Solution:

(a) $V_{av} = V_o = \gamma V_d$

$$\gamma = \frac{V_{av}}{V_d} = \frac{50}{100} = 0.5$$

(b) T= 1/ f = 1 / 1000 = 1ms

$$\gamma = \frac{t_{on}}{T}$$

$t_{on} = \gamma T = 05 \text{ x}1\text{ms} = 0.5 \text{ ms}$.

(c) $V_{orms} = \sqrt{\gamma}\ V_i = \sqrt{0.5}\ x\ 100 = 70.07\ V$

(d)

$$I_{av} = \frac{V_{av}}{R} = \frac{50}{10} = 5\ A$$

(e)

$$I_{max} = \frac{V_{av}}{R} + \frac{t_{off}}{2L} V_{av} = \frac{50}{10} + \frac{(1 - 0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} x50$$

= 5 + 0.25 = 5,25 A

$$I_{min} = \frac{V_{av}}{R} - \frac{t_{off}}{2L} V_{av} = \frac{50}{10} - \frac{(1 - 0.5) \times 10^{-3}}{2 \times 50 \times 10^{-3}} x50$$

= 5 - 0.25 = 4.75 A

(f)

$$I_{s(av)} = \frac{\gamma}{2} (I_{min} + I_{max}) = \gamma I_{av} = 0.5\ \times 5 = 2.5A$$

$$P_{in} = I_{s(av)} V_d = 2.5\ \times 100 = 250\ W$$

(g) $I_{p-p} = \Delta I = I_{max} - I_{min} = 5.25 - 4.75 = 0.5A$

9.4 Application to DC Drives

Chopper-Fed DC Motor Drive

- **D.C motor** is considered a SISO (Single Input and Single Output) system having torque/speed characteristics compatible with most mechanical loads. This makes a **DC motor** controllable over a wide range of speeds by proper adjustment of the terminal voltage.
- A simple chopper – fed DC motor drive is shown in Fig.9.8. The basic principle behind DC motor speed control is that the output speed of DC motor can be varied by controlling armature voltage for speed below and up to rated speed keeping field voltage constant. The armature voltage can be controlled by controlling the duty cycle of the converter (here the converter used is a DC chopper).

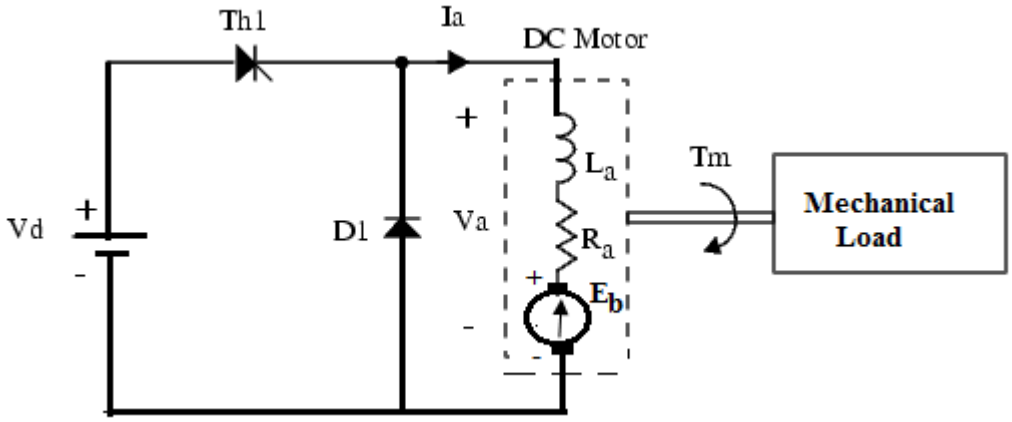


Fig.9.8: Chopper-Fed DC Motor Drive.

- In Fig. 9.8, the converter output gives the dc output voltage V_a required to drive the motor at the desired speed. In this diagram, the DC motor is represented by its equivalent circuit consisting of inductor La and resistor Ra in series with the back emf (E_b) .The thyristor Th1 is triggered by a pulse width modulated (PWM) signal to control the average motor voltage. Theoretical waveforms illustrating the chopper operation are shown in Fig.9.9.

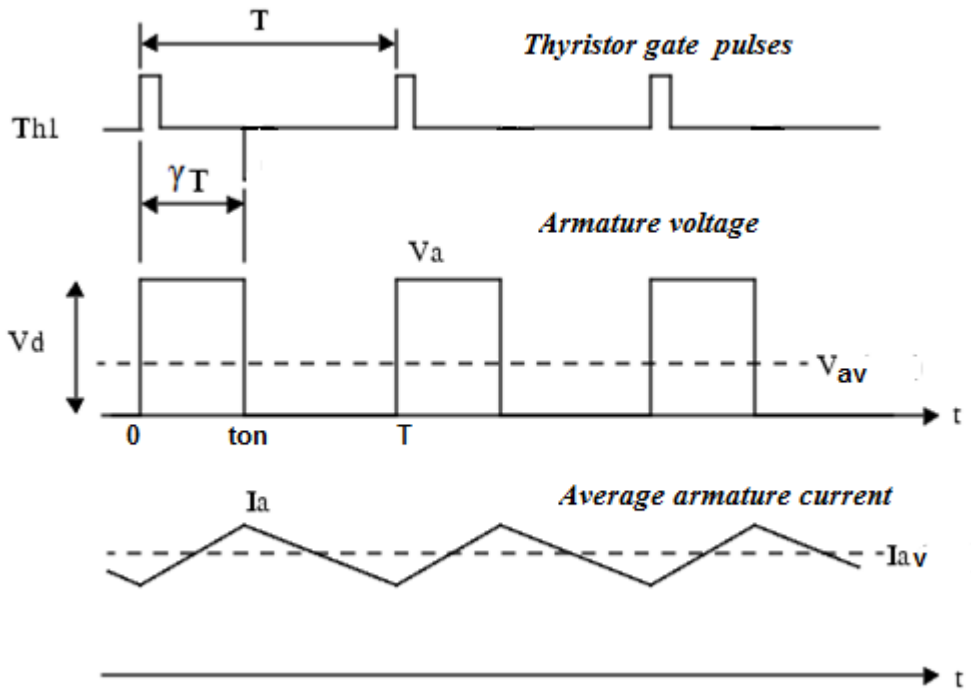


Fig.9.9

The average armature voltage is a direct function of the chopper duty cycle γ ,

$$V_{av} = \gamma V_d$$

Note that this relation is valid only when the armature current is continuous. In steady state, the armature average current is equal to

$$I_a = \frac{V_{av} - E_b}{R_a}$$

Where I_a = average armature current (A).

E_b is the internal generated voltage (back emf) is given by:

$$E_b = K_e \varphi n$$

Finally, solving for the motor's speed:

$$n = \frac{V_{av}}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

$$or \quad n = \frac{\gamma V_d}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

Where n = speed in rpm.

and the motor torque is

$$T_d = K_T I_a \varphi$$

K_T = Torque constant = 9.55 K_e (See Lecture No.8),

At starting, $n = 0$. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e \emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st} = \frac{9.55\, \gamma V_d}{R_a} K_e \emptyset$$

The speed – torque characteristics of a separately excited d.c. motor is shown in Fig. 12.3 for different values of the duty cycle γ. It is clear that , as the duty cycle is reduced, the no load speed and the starting torque are reduced accordingly and the characteristic lines shift down ward in parallel manner.

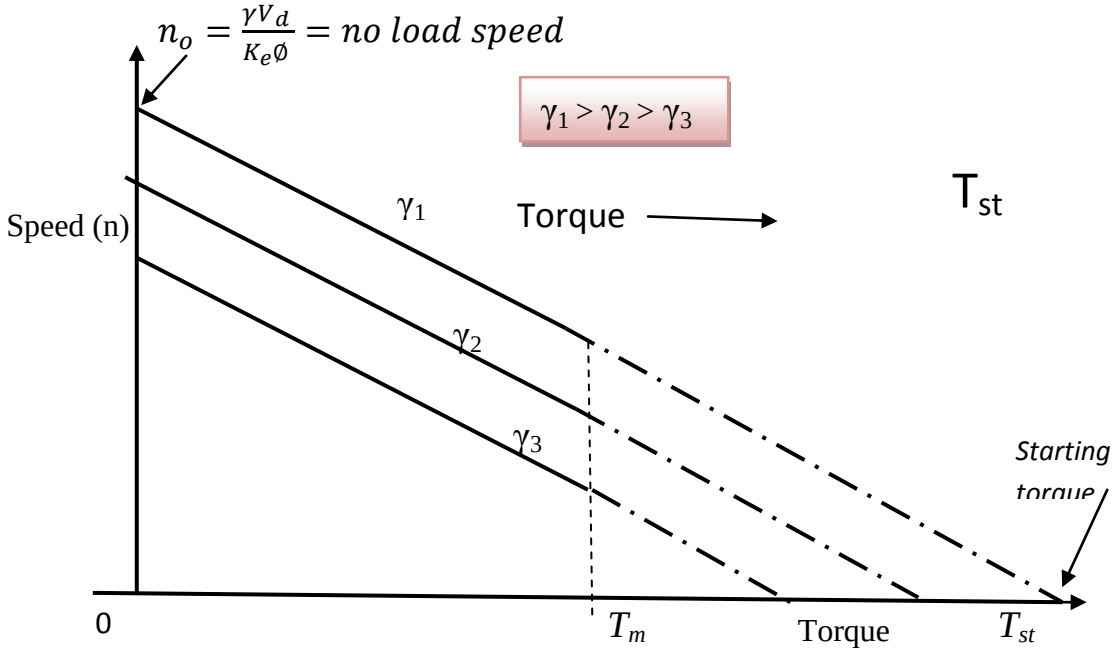


Fig.9.10 Speed – torque characteristics of a d.c. motor with d.c. chopper drive.

Example 4

A separately excited d.c. motor with R_a = 1.2 ohms and L_a = 30 mH , is to be controlled using class-A thyristor chopper as shown in Fig.9.11 .The d.c. supply V_d = 120V . By ignoring the effect of the armature inductance L_a , it is required to:

(a) Find the no load speed and starting torque of the motor when the duty cycle γ =1.

(b) Draw the speed torque characteristics for the motor when the duty cycle γ = 1. The motor design constant $Ke\Phi$ has a value of 0.042 V/rpm.

(c) Find the speed of the motor n (rpm) when a torque of 8 Nm is applied on the motor shaft and the duty cycle is set to γ = 0.5.

Solution

The average armature voltage is

$$V_{av} = \gamma V_d = 1x\,120 = 120\,V$$

The motor's speed:

$$n = \frac{V_{av}}{K_e\emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_d$$

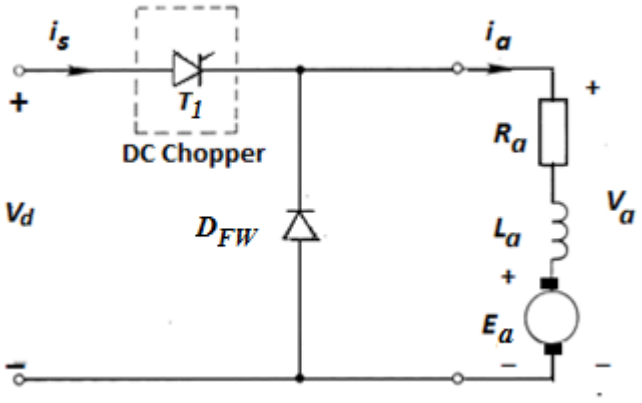


Fig. 9.11 Thyristor chopper drive.

At no load T_d = 0, hence

or $$n_o = \frac{\gamma V_d}{K_e\emptyset} = \frac{120}{0.042} = 2857\ rpm$$

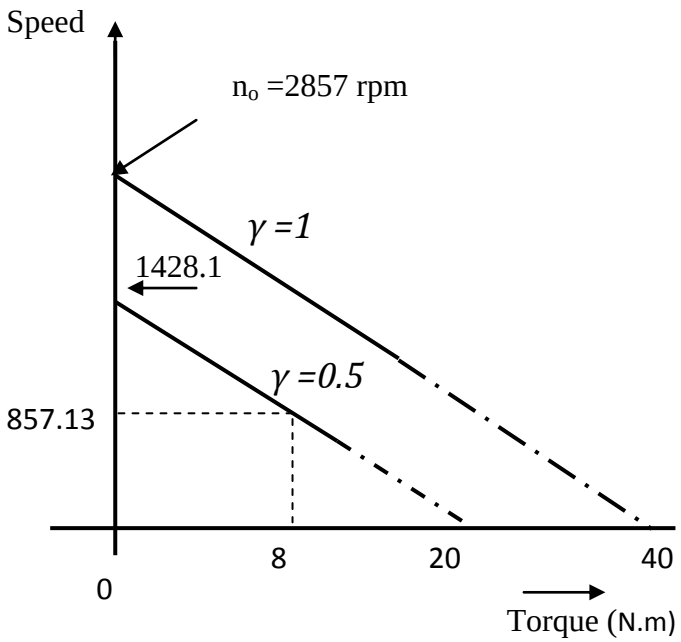
At starting, n = 0. The starting torque T_{st} may be found as:

$$n = 0 = \frac{\gamma V_d}{K_e\emptyset} - \frac{R_a}{K_T K_e \emptyset^2} T_{st}$$

$$\therefore T_{st} = \frac{9.55\,\gamma V_d}{R_a} K_e\emptyset$$

$$T_{st} = \frac{9.55\,x120}{1.2} x0.042 = 40\ N.m$$

Fig.9.11 Speed-torque characteristics



(b) At $\gamma = 0.5$

$$V_a = \gamma V_d = 0.5\ x\ 120 = 60\ V$$

$$n_o = \frac{\gamma V_d}{K_e \emptyset} = \frac{60}{0.042} = 1428.5\ rpm$$

$$T_{st} = \frac{9.55\ x60}{1.2} x0.042 = 20\ \text{N.m}$$

At $\gamma = 0.5$, $T_L = 8$ N.m

$$n = \frac{60}{0.042} - \frac{1.2}{9.55(0.042)^2} x8 = 857.13\ \text{rpm}$$

Note: K_T = Torque constant = 9.55 K_e

Example 5

In the microcomputer -controlled class –A IGBT transistor DC chopper shown in Fig.12.6, the input voltage V_d = 260V, the load is a separately excited d.c. motor with Ra = 0.28 Ω and L_a = 30 mH . The motor is to be speed controlled over a range 0 – 2500 rpm , provided that the load torque is kept constant and requires an armature current of 30A .

(a) Calculate the range of the duty cycle γ required if the motor design constant $K_e\Phi$ has a value of 0.10 V/rpm.

(b) Find the speed of the motor *n* (rpm) when the chopper is switched fully ON such that the duty cycle γ = 1.0.

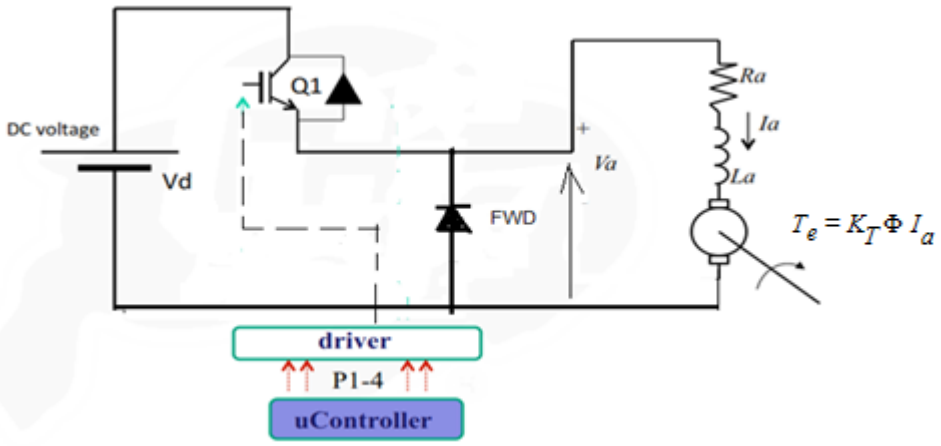


Fig.12.6 IGBT Chopper drive.

Solution

(a) With steady – state operation of the motor, the armature inductance L_a behaves like a short circuit and therefore has no effect at all.

At stand still n = 0 , and therefore E_a = 0 , hence from Eq.(12.22)

$$I_a = \frac{V_a - E_a}{R_a} = \frac{V_{a0} - 0}{0.28} = 30\ A$$

$$\therefore\ V_{a0} = 0.28x30 = 8.4\ V$$

At full speed n = 2500 rpm ,

$$E_{a2500} = K_e\ \phi\ n = 0.1\ \times\ 2500 = 250\ V$$

For separately excited d.c. motor,

$$V_{a2500} = E_a + I_a R_a = 250 + 30 \times 0.28 = 258.4\ V$$

Therefore the range of the duty cycle γ will be:

$$\gamma_0 = \frac{V_{a0}}{V_d} = \frac{8.4}{260} = 0.0323$$

Similarly

$$\gamma_{2500} = \frac{V_{a2500}}{V_d} = \frac{258.4}{260} = 0.9938$$

(b) When the chopper is switched fully on, i.e. γ =1, then$V_a = V_d = 260\ V$.
At this condition,

$$V_a|(\gamma = 1) = E_a + I_a R_a = K_e\ \phi\ n + I_a R_a\ = 260$$

$$0.1\ n + 30\ x\ 0.28\ = 260 \qquad \rightarrow \quad n = 2516 \text{ rpm}$$

10. AC-TO-AC Converters

10.1 Single-phase AC voltage controller

A single – phase AC voltage controller is shown in Fig.10.1.In this controller (some times called AC chopper), two thyristors (SCRs) are connected in inverse-parallel (back to back) to perform the function of an electronic switch suitable for use with a.c supply. If suitable gating pulses are applied to the thyristors while their respective anode voltage is positive, current conduction is initiated. The conduction angle depends on the triggering-angle, which is measured form anode voltage zero.

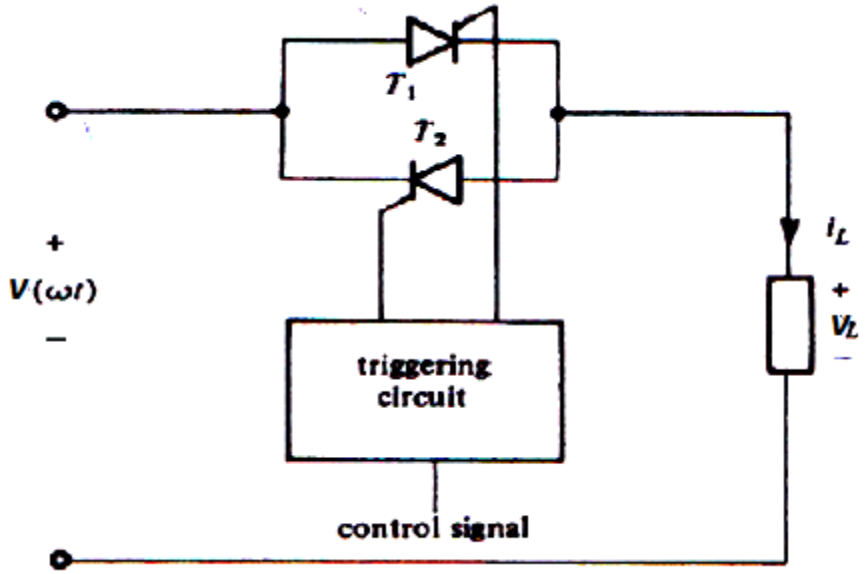


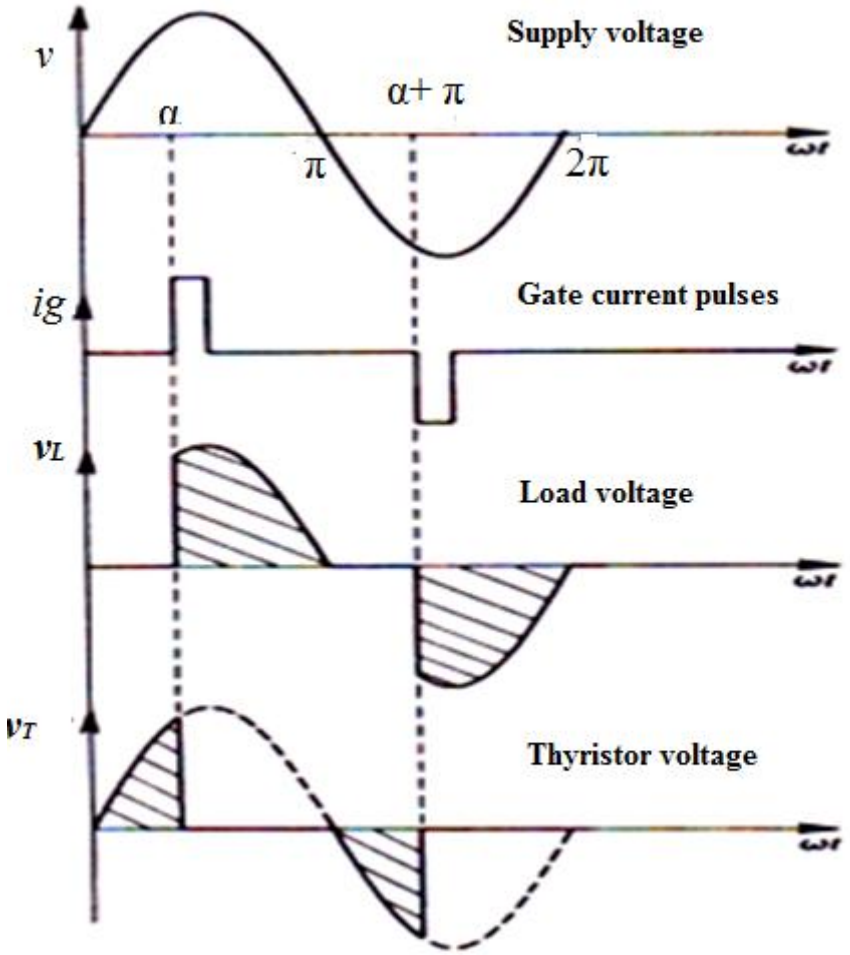
Fig.10.1 basic circuit arrangement of single-phase inverse –parallel connected thyristor pair.

Two different modes of switching operation are mainly used:

- If $\alpha_1 = \alpha_2 = \alpha$, i.e. the two thyristors are triggered at same angle value, then the triggering mode is called **symmetrical** angle triggering.
- If triggering is used with two different angles $\alpha_1 \neq \alpha_2$ then the triggering mode is called **asymmetrical** angle triggering.

10.1.2. Resistive load with symmetrical phase-angel triggering

With resistive load single gating pulse of magnitude 1-3 v is usually sufficient to switch on an SCR .Waveforms for sinusoidal supply voltage with an arbitrary triggering angle α=60º is given in fig.10.2.



ω

Fig10.2: Theoretical voltage waveforms for the circuit of fig.1, for R- load and α = 60º.

The voltage waveforms are defined in terms of the peak supply phase voltage V_m by

- The supply voltage *v*

$$v = V_m sin\omega t \qquad (1)$$

- The load voltage v_L

$$v_L = V_m sin\omega t \begin{cases} \pi \quad ,2\pi, \dots \\ \alpha, \pi + \alpha, \dots \end{cases}$$

$$v_L = 0 \qquad elsewhere \qquad (2)$$

- The thyristor voltage v_T

$$v_T = V_m sin\omega t \begin{cases} \alpha \quad ,\pi + \alpha, \dots \\ 0 \quad\quad ,\pi, \dots \end{cases} \qquad (3)$$

10.1.3 R.M.S. values of the load voltage and current.

Any function $v_L(wt)$ that is periodic in 2π radians has a root mean square (r.m.s) or effective value defined by:

$$V_L = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_L^2\,(\omega t) d\omega t} \qquad (4)$$

The function $v_L(wt)$ is defined by (2) for the circuit of fig.10.1
Substituting (2) into (4) gives:

$$V_L = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\pi}(V_m sin\omega t)^2\, d\omega t + \frac{1}{2\pi}\int_{\alpha+\pi}^{2\pi}(V_m sin\omega t)^2\, d\omega t}$$

But

$$\frac{1}{2\pi}\int_{\alpha}^{\pi}(V_m sin\omega t)^2\, d\omega t = \frac{1}{2\pi}\int_{\alpha+\pi}^{2\pi}(V_m sin\omega t)^2\, d\omega t$$

We can say that

$$V_L = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}(V_m sin\omega t)^2\, d\omega t}$$

$$V_L = \sqrt{\frac{V_m^2}{\pi}\int_{\alpha}^{\pi}\frac{1}{2}(1 - cos2\omega t)\, d\omega t}$$

$$v_L = V_m\sqrt{\frac{1}{2\pi}\{(\pi - \alpha) + \frac{sin2\alpha}{2}\}}$$

The r.m.s vale of the sinusoidal supply voltage is given by the standard relationship

$$V_s = \frac{V_m}{\sqrt{2}}$$

The r.m.s load voltage V_L can therefore re-written as

$$V_L = V_s\sqrt{\frac{1}{2\pi}\{2(\pi - \alpha) + sin2\alpha\}}$$

With resistive load the instantaneous current is given by:

$$i = \frac{v_L}{R} = \frac{V_m}{R} sin\omega t \begin{cases} \pi \quad ,2\pi, \dots \\ \alpha, \pi + \alpha, \dots \end{cases}$$

The r.m.s load current I can therefore by Written directly from:

$$I_L = \frac{V_m}{R} \sqrt{\frac{1}{2\pi}\{ (\pi - \alpha) + \frac{sin2\alpha}{2} \}}$$

In terms of the r.m.s supply voltage *Vs* the r.m.s current is given by:

$$I_L = \frac{V_s}{R} \sqrt{\frac{1}{2\pi}\{ 2(\pi - \alpha) + sin2\alpha \}}$$

10.1.4 Power and power factor

- Average Power delivered to the load

For resistive load, the average load power P_L is

$$P_L = I^2 R \qquad or \qquad P_L = \frac{V^2}{R}$$

$$P_L = \frac{V_s^2}{R}\left[\frac{1}{2\pi}\{ 2(\pi - \alpha) + sin2\alpha \} \right]$$

- Apparent power taken from the supply

$$P_s = V_s \,.\, I_L = \frac{V_s^2}{R} \sqrt{\frac{1}{2\pi}\{ 2(\pi - \alpha) + sin2\alpha \}}$$

- Input power factor

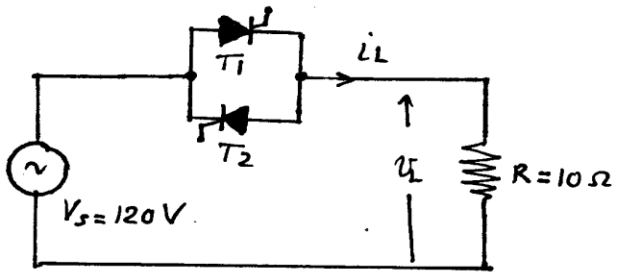
$$PF = \frac{Average\ power}{Apparent\ power}$$

$$PF = \frac{V_L . I_L}{V_S . I_L} = \frac{V_L}{V_S} = \sqrt{\frac{1}{2\pi}\{\, 2(\pi - \alpha) + sin2\alpha \,\}}$$

Home work: Explain why we have a power factor while the load is pure resistance?

AC-CHOPPERS

Example 2: A single-phase full-wave ac chopper shown in Fig. A. has a resistive load of $R = 10\Omega$ and the input voltage is $V_s = 120V$ (rms), 50Hz. The firing angles of Thyristors T_1 and T_2 are equal: $\alpha_1 = \alpha_2 = \alpha = \frac{\pi}{2}$. Determine (a) the rms output voltage $V_{Lr.m.s}$, (b) the input power factor and (c) the average and r.m.s values of the thyristor currents.



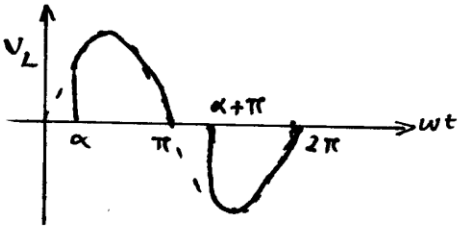
Solution

(a) $V_m = \sqrt{2}V_s$

$$V_{Lr.m.s} = \left\{\frac{1}{\pi}\int_{\alpha}^{\pi}(\sqrt{2}V_s)^2 \sin^2\omega t\, d\omega t\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{V_s^2}{\pi}\int_{\alpha}^{\pi}(1-\cos 2\omega t)\, d\omega t\right\}^{\frac{1}{2}}$$

$$= V_s\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$



For $\alpha = \frac{\pi}{2}$

$$V_{Lr.m.s} = 120\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin\pi}{2}\right)\right]^{\frac{1}{2}} = \frac{120}{\sqrt{2}}$$

$$= 84.85V.$$

(b) The r.m.s value of the load current is

$$I_{Lr.m.s} = \frac{V_{Lr.m.s}}{R} = \frac{84.85}{10} = 8.485\ A.$$

Load power is

$$P_L = I_{L.r.m.s}^2 \cdot R = (8.485)^2 \times 10 = 719.95\ W.$$

Because the input current is the same as the load current, the input VA rating is:

$$VA = V_s I_s = V_s I_{Lr.m.s} = 120 \times 8.485 = 1018.2W$$

The input PF is

$$PF = \frac{P_L}{VA} = \frac{V_{Lr.m.s} \cdot I_{Lr.m.s}}{V_s \cdot I_{Lr.m.s}} = \frac{V_{Lr.m.s}}{V_s}$$

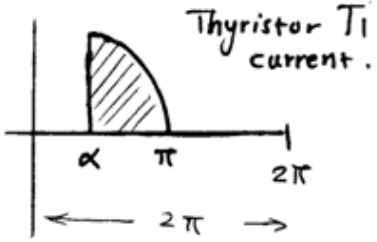
$$= \frac{84.85}{120} = 0.707 \quad (\text{lagging}). \quad [\text{WHY?}]$$

c. The average thyristor current

$$I_{T(ave)} = \frac{1}{2\pi R} \int_{\alpha}^{\pi} V_m \sin\omega t \, d\omega t$$

$$= \frac{\sqrt{2} V_s}{2\pi R} (\cos\alpha + 1)$$

$$= \frac{\sqrt{2} \times 120}{2\pi \times 10} = 2.7 A .$$



$$i_{T1} = I_m \sin\omega t$$
$$= \frac{V_m}{R} \sin\omega t .$$

- The r.m.s. value of the thyristor current

$$I_{T(r.m.s)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} \left[\frac{\sqrt{2} V_s}{R} \sin\omega t\right]^2 d\omega t}$$

$$= \sqrt{\frac{2V_s^2}{4\pi R^2} \int_{\alpha}^{\pi} (1 - \cos 2\omega t) \, d\omega t}$$

$$= \frac{V_s}{\sqrt{2} R} \left[\frac{1}{\pi} \left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{\frac{1}{2}}$$

$$= \frac{120}{2 \times 10} = 6 A$$

10.2 Harmonics analysis of the load voltage waveform of AC chopper

The load voltage v_L waveform shown in Fig.10.3 is a nonsinusoidal waveform that contains harmonic components.

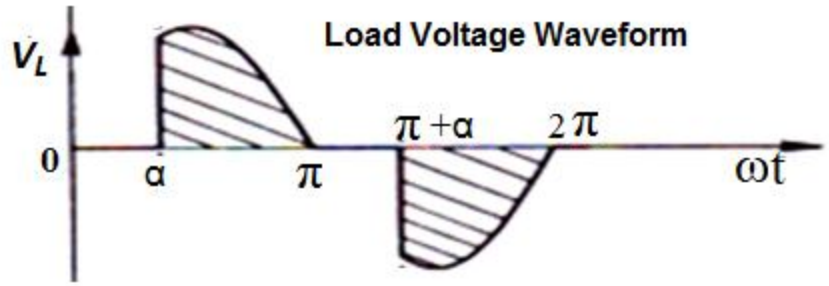


Fig.10.3 Load voltage waveform of a single – phase ac chopper with resistive load.

In terms of harmonics, this waveform can be expressed in Fourier series as:-

$$v_L(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \psi_n)$$

where, the Fourier coefficients are

$$\frac{a_0}{2} = \frac{1}{2\pi}\int_0^{2\pi} v_L(\omega t)\, d\omega t$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t) \cos n\omega t\, d\omega t$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t) \sin n\omega t\, d\omega t$$

n= the n^{th} order harmonics (n=1,2,3,…….)

C_n = the amplitude of the n[th] order harmonics

$$C_n = \sqrt{a_n^2 + b_n^2}$$

ψ_n = phase angle of the n[th] harmonic component given as

$$\psi_n = \tan^{-1}\frac{a_n}{b_n}$$

For the fundamental component (n=1):-

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\cos\omega t\, d\omega t$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\sin\omega t\, d\omega t$$

$$c_1 = \sqrt{a_1^{\,2} + b_1^{\,2}} = \text{peak value of fundamental component}$$

$$\psi_1 = \tan^{-1}\frac{a_1}{b_1} = \text{displacement angle between the fundamental and the datum point.}$$

For the fundamental of the load voltage,

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\cos\omega t\, d\omega t$$

$$= \frac{V_m}{2\pi}(\cos 2\alpha - 1)$$

$$b_1 = \frac{V_m}{2\pi}(2(\pi - \alpha) + \sin 2\alpha)$$

$$c_1 = \frac{V_m}{2\pi}\sqrt{(\cos 2\alpha - 1)^2 + [2(\pi - \alpha) + \sin 2\alpha]^2}$$

$$\psi_1 = \tan^{-1}\frac{a_1}{b_1} = \tan^{-1}\frac{\cos 2\alpha - 1}{2(\pi - \alpha) + \sin 2\alpha}$$

For the nth Fourier harmonic component.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\cos n\omega t\, d\omega t$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\sin n\omega t\, d\omega t$$

$$a_n = \frac{V_m}{2\pi}\left[\begin{matrix}\frac{1}{n+1}\{1+(-1)^{n+1}\}\{\cos(n+1)\,\alpha - 1\} \\ -\frac{1}{n-1}\{1+(-1)^{n-1}\}\{\cos(n-1)\,\alpha - 1\}\end{matrix}\right]$$

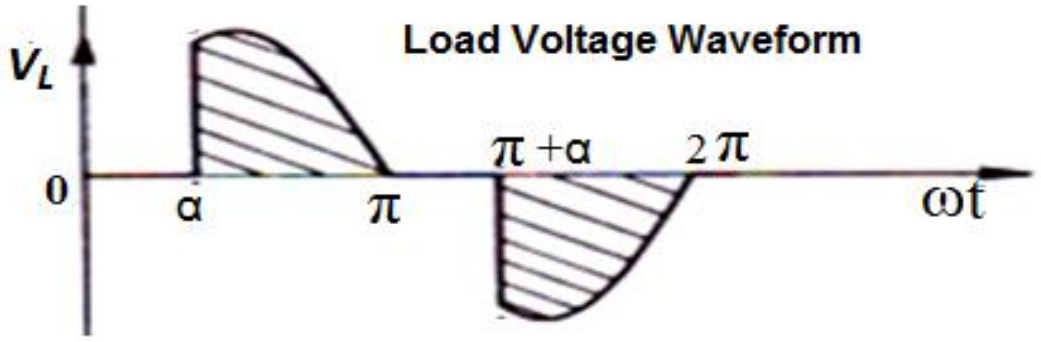
$$b_n = \frac{V_m}{2\pi}\left[\begin{matrix}\frac{1}{n+1}\{1+(-1)^{n+1}\}\{\sin(n+1)\,\alpha - 1\} \\ -\frac{1}{n-1}\{1+(-1)^{n-1}\}\{\sin(n-1)\,\alpha - 1\}\end{matrix}\right]$$

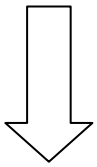
- For even order harmonic terms (n= 2,4,6,…….) the values of $(-1)^{n+1}$ and $(-1)^{n-1}$ are (-1) , hence $a_n = 0$ and $b_n = 0$ (no even harmonics).
- For odd harmonic terms (n=3,5,7,…….) the values of $(-1)^{n+1}$ and $(-1)^{n-1}$ are unity. Hence $a_n \neq 0$ and $b_n \neq 0$.

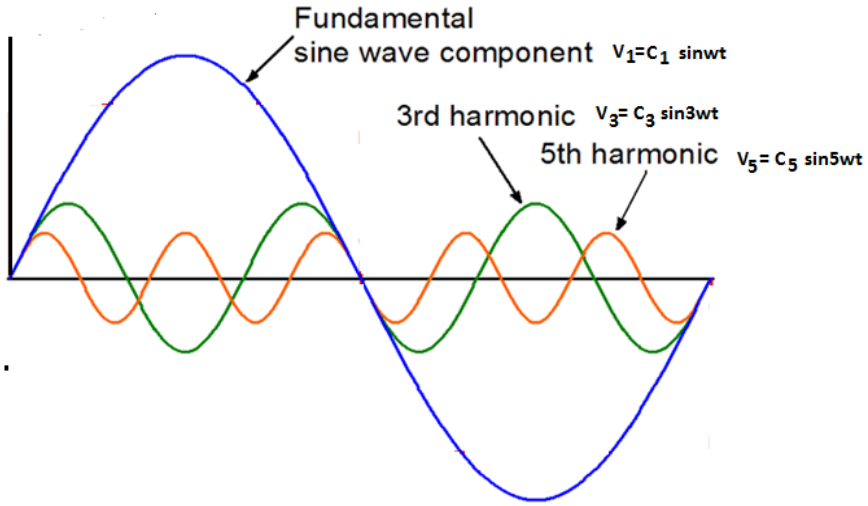
$$a_n = \frac{V_m}{2\pi}\left[\begin{matrix}\frac{2}{n+1}\{\cos(n+1)\,\alpha - 1\} \\ -\frac{2}{n-1}\{\cos(n-1)\,\alpha - 1\}\end{matrix}\right]$$

$$b_n = \frac{V_m}{2\pi}\left[\begin{matrix}\frac{2}{n+1}\{\sin(n+1)\,\alpha - 1\} \\ -\frac{2}{n-1}\{\sin(n-1)\,\alpha - 1\}\end{matrix}\right]$$

Hence only odd harmonics , 3^{rd} ,5^{th} ,7^{th} , 9^{th} ,11^{th} …etc are exist in the load voltage waveform.







The variations of these harmonics with the triggering angle α is shown in Fig.10.4.

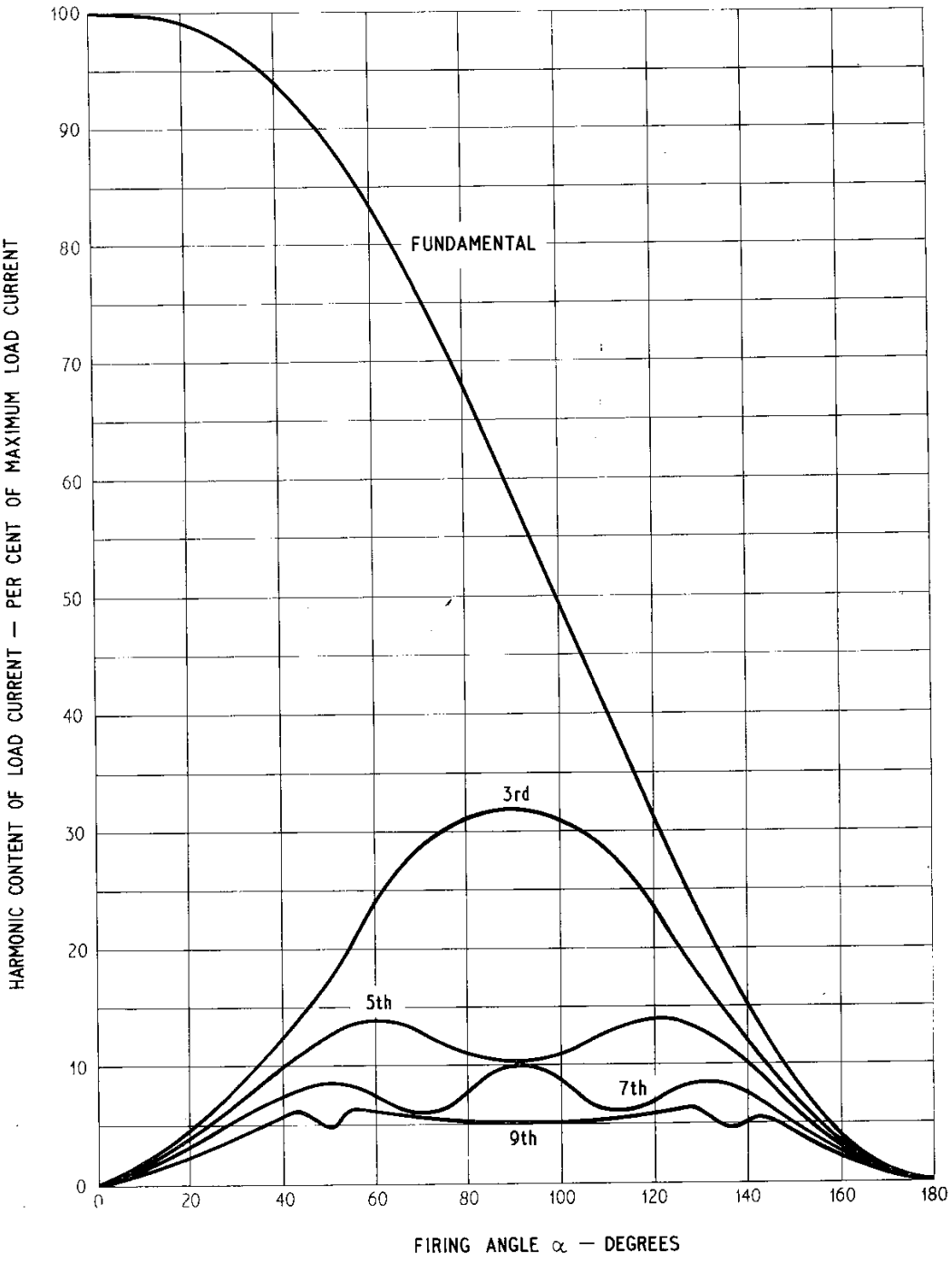


Fig.10.4 Variation of harmonic amplitude with the triggering angle α.

10.3 Calculation of power dissipation in terms of harmonics

In general the average value of the power is calculated as:

$$P_L = \frac{1}{2\pi}\int_0^{2\pi} v_L \; i \; dt$$

$$= I_L{}^2 \; R = \frac{V_L{}^2}{R} \text{ where } I_L \;\&\; V_L \text{ are the r.m.s. values}$$

$$= \frac{V^2}{2\pi R}[2(\pi - \alpha) + \sin 2\alpha]$$

In terms of harmonic components:-

$$P_L = R(I_1{}^2 + I_3{}^2 + I_5{}^2 + ...) = \frac{1}{R}(V_{L1}^2 + V_{L3}^2 + V_{L5}^2 + ...)$$

In terms of fundamental components:

$$P_L = V_s \; I_1 \cos \psi_1$$

where

$$I_1 = \text{r.m.s. fundamental current} = \frac{v_1}{\sqrt{2}}\frac{1}{R}$$

$$\cos\psi_1 = \frac{b_1}{c_1}$$

10.4 Power Factor in systems with sinusoidal voltage (at supply) but non-sinusoidal current

In general,

$$PF = \frac{\text{Average Power}}{\text{Apparent Voltamperes}} = \frac{P_L}{V_s \; I_s}$$

V_s = r.m.s. voltage at supply.

I_s = r.m.s. current at supply.

$i(wt)$ is periodic in 2π but is non-sinusoidal.

Average power is obtained by combining in-phase voltage and current components of the same frequency.

$$P = Vs\ I_1 \cos\psi_1$$

$$PF = \frac{P}{Vs\ Is} = \frac{Vs\ I_1 \cos\psi_1}{Vs\ Is} = \frac{I_1}{Is}\cos\psi_1$$

$$= \text{Distortion Factor x Displacement Factor}$$

where $Distortion\ factor = \frac{I_1}{I_s}$

Displacement factor = $\cos\Psi_1$

- **Distortion Factor = 1 for sinusoidal operation**
- **Displacement factor is a measure of displacement between *v*(ω *t*) and *i*(ω*t*).**
- **Displacement Factor =1 for sinusoidal resistive operation.**

Calculation of PF

$$PF = \frac{I^2 R}{VI} = \frac{\frac{V^2 R}{2\pi R^2}[2(\pi-\alpha)+\sin 2\alpha]}{V\frac{V}{R}\sqrt{\frac{1}{2\pi}[2(\pi-\alpha)+\sin 2\alpha]}}$$

$$= \sqrt{\frac{1}{2\pi}[2(\pi-\alpha)+\sin 2\alpha]}$$

Example 10.2: In a single- phase resistive circuit in which the load voltage is controlled by symmetrical phase angle triggering of a pair of inverse- parallel connected thyristors. If the supply voltage is vs = 200 sinωt , α =60˚ and R= 50Ω , calculate :

(a) The rms value of the fundamental component of the load current,

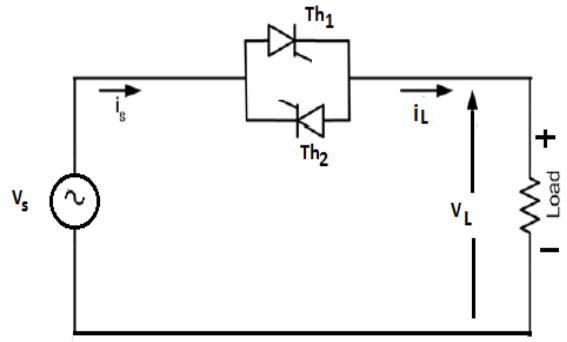
(b) The value of the displacement angle φ_1 between the supply voltage and the fundamental component of the load current,

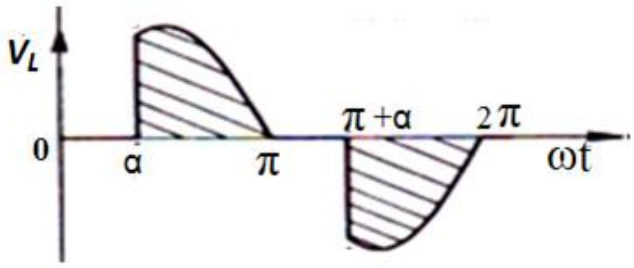
(c) The rms value of the load voltage and the power factor of the circuit.

(d) The displacement factor and the distortion factor.

Solution:

The circuit and the output voltage waveform are shown in figure below:





Circuit Output voltage waveform

(a) The peak value of the fundamental component of the load voltage is found as,

$$C_1 = \frac{V_m}{2\pi}\sqrt{(cos2\alpha-1)^2+[2(\pi-\alpha)+sin2\alpha]^2}$$

Vm = 200V, α = 60˚ = π/3

$$C_1 = \frac{200}{2\pi}\sqrt{(cos2x60-1)^2+[2\left(\pi-\frac{\pi}{3}\right)+sin2x60]^2}$$

$$C_1 = 167.8\ V$$

The rms value of the fundamental component of load voltage V_{L1} is calculated as,

$$V_{L1} = \frac{C_1}{\sqrt{2}} = \frac{167.8}{\sqrt{2}} = 118.7\ V$$

The fundamental component of the load current is,

$$I_{L1} = \frac{V_{L1}}{R} = \frac{118.7}{50} = 2.37\ A$$

(b) The displacement angle ψ_1

$$\psi_1 = tan^{-1}\frac{a_1}{b_1} = tan^{-1}\frac{cos2\alpha - 1}{2(\pi - \alpha) + sin2\alpha}$$

$$\psi_1 = tan^{-1}\frac{cos2x60 - 1}{2\left(\pi - \frac{\pi}{3}\right) + sin2x60} = -16.5^o$$

(c) The rms value of the load voltage is given by ,

$$v_L = V_m\sqrt{\frac{1}{2\pi}\{(\pi - \alpha) + \frac{sin2\alpha}{2}\}}$$

$$v_L = 200\sqrt{\frac{1}{2\pi}\{\left(\pi - \frac{\pi}{3}\right) + \frac{sin120^o}{2}\}} = 126.85\ V$$

$$PF = \frac{V_L\,.\,I_L}{V_s\,.\,I_L} = \frac{V_L}{V_s} = \sqrt{\frac{1}{2\pi}\{\,2(\pi - \alpha) + sin2\alpha\,\}}$$

$$PF = \frac{V_L\,.\,I_L}{V_s\,.\,I_L} = \frac{V_L}{V_s} = \frac{126.85}{200/\sqrt{2}} = 0.897$$

$$or: \quad PF = \sqrt{\frac{1}{2\pi}\{\,2\left(\pi - \frac{\pi}{3}\right) + sin120^o\,\}} = 0.897$$

(d) The displacement factor : $cos\psi_1 = \cos(-16.5) = \cos 16.5 = 0.9588$

The distortion factor : $DF = \frac{I_{L1}}{I_L} = \frac{2.37}{126.85/50} = 0.934$

PF can also be calculated from : PF = displacement factor x distortion factor

= 0.9588 x 0.934 = 0.897

11. Single-phase AC Controller with R-L load

The single –phase AC controller with resistive load is shown in Fig.11.1.Due to the inductance in the circuit , the current in thyristor T1 would not fall to zero at ωt = π, when the source voltage v_s start to be negative. Thyristor T1 continues to conduct until its current i1 falls to zero at ωt =ß.

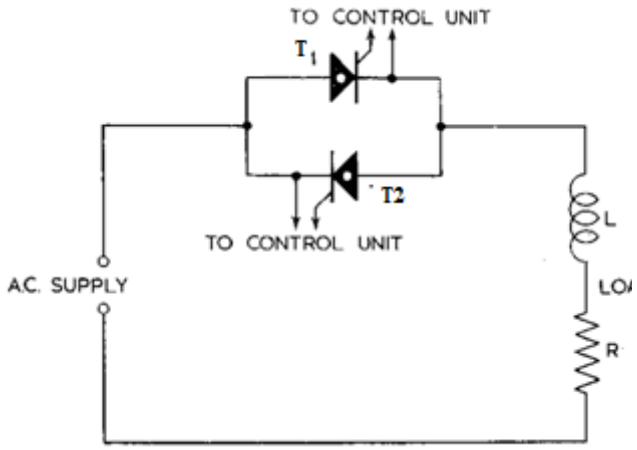
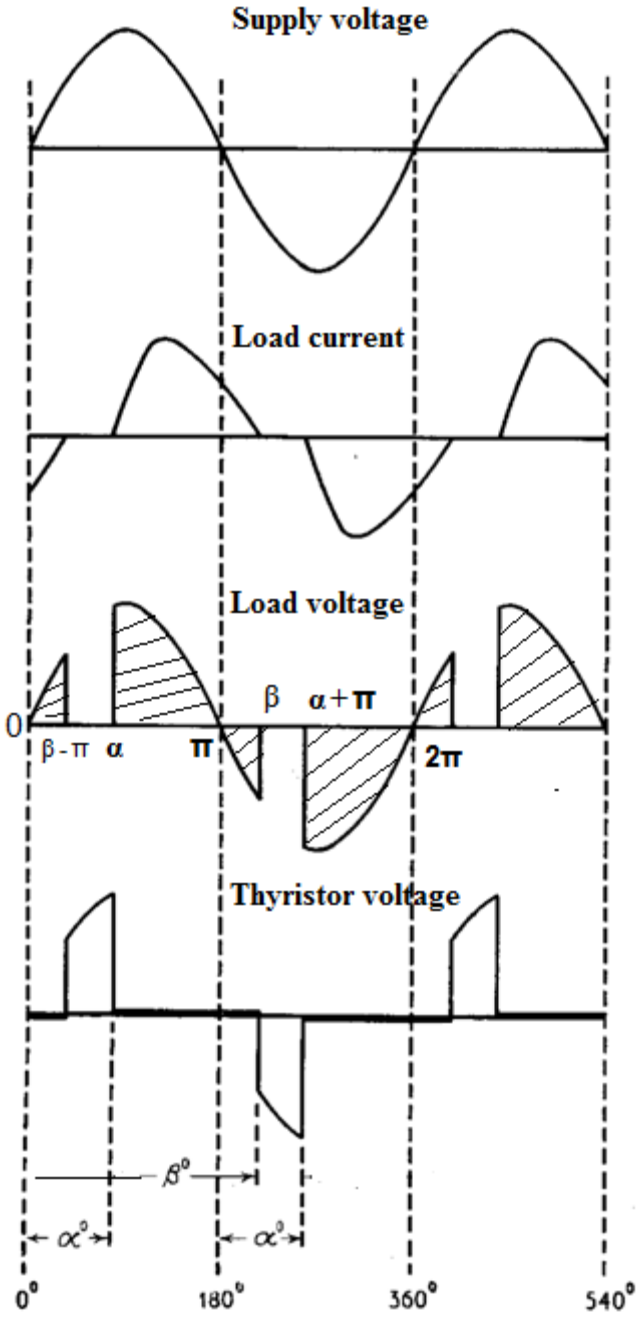


Fig.1



By KVL;

$$v_s = v_o = V_R + v_L$$

The equation for the current through R-L load can be found from the solution of the differential equation:

$$L\frac{di}{dt} + i\,R = V_m \sin \omega t$$

The solution of this differential equation is:

- The start of conduction is delayed until $\omega t = \alpha$. Subsequent to triggering, let the instantaneous current $i(\omega t)$ consists of hypothetical steady-state components $i_{ss}(\omega t)$ and transient component $i_{trans}(\omega t)$,

$$\therefore i(\omega t) = i_{ss}(\omega t) + i_{trans}(\omega t)$$

Now at $\omega t = \alpha$, the instantaneous steady-state component has the value,

$$i_{ss}(\alpha) = \frac{V_m}{Z}\sin(\alpha - \theta)$$

where

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

The transient component of the current,

$$i_{trans} = Ae^{-\frac{R}{L}t}$$

Hence the equation for the current *i*,

$$i(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\frac{R}{L}t} \quad \alpha \le \omega t \le \beta$$

The constant A can be found from the initial conditions:

At $\omega t = \alpha$, $i = 0$, hence from eq.(2) ,

$$A = -\frac{V_m}{Z}\sin(\alpha - \theta)e^{-(\frac{R}{L})(\frac{\alpha}{\omega})}$$

- Subsequent to α

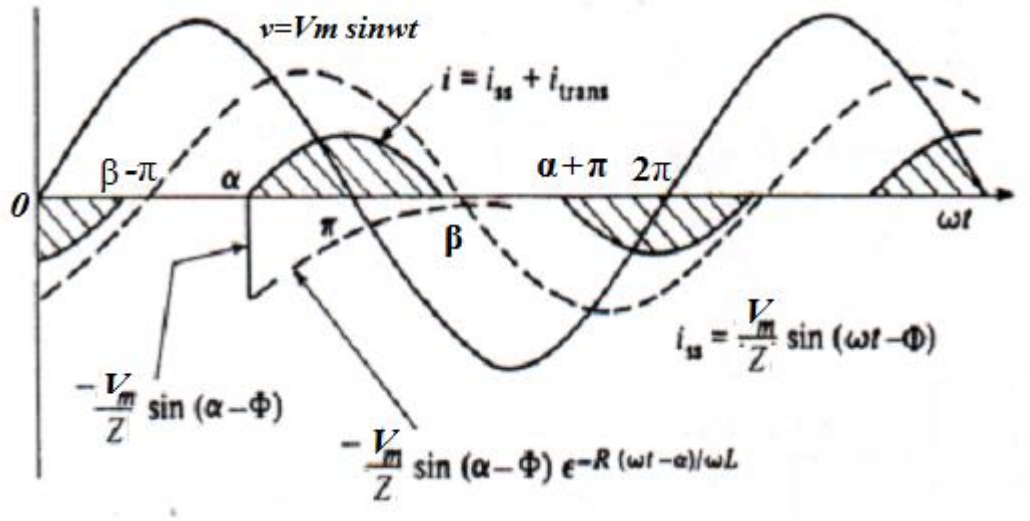
The transient component decay exponentially from it's instantaneous value of [$-\frac{V_m}{Z}\sin(\alpha - \theta)$] by the time constant $\tau = \frac{L}{R.}$. Thus

$$i_{trans}(\omega t) = \frac{-V_m}{Z}\sin(\alpha - \theta)e^{-\frac{t}{\tau}}$$

For $\omega t > \alpha$, $$i_{trans}(\omega t) = -\frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$

- The complete solution for the current at first cycle

$$i(\omega t) = \frac{v_m}{Z}\sin(\omega t - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$



Extinction Angle β

For the current in the interval $\alpha \le \omega t \le \beta$

$$i(\omega t) = \frac{v_m}{Z}\sin(\omega t - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\omega t - \alpha)}$$

But at $\omega t = \beta, i(\omega t) = 0$, hence

$$0 = \frac{v_m}{Z}\sin(\beta - \theta) - \frac{V_m}{Z}\sin(\alpha - \theta)e^{-\frac{R}{\omega L}(\beta - \alpha)}$$

But $\frac{R}{\omega L} = \cot\theta$

$$\therefore 0 = \sin(\beta - \theta) - \sin(\alpha - \phi)e^{-\cot\theta(\beta - \alpha)}$$

If α and θ are known, β can be calculated. However, this is a transcendental equation (i.e. cannot be solved explicitly and no way of obtaining $\beta = f(\alpha, \theta)$).

Method of solution is by iteration,

e.g. If $\theta = 60^0$, $\alpha = 120^0 = 2\pi / 3$

$$\cot\theta = 0.578$$

$$\sin(\beta - 60^0) = \sin(120^0 - 60)e^{-0.578(\beta - \frac{2\pi}{3})} \Rightarrow \beta = 222^0$$

Approximate solution of β :-

$$\beta = 180^0 + \theta - \Delta$$

where

$$\Delta = 5^0 \sim 10^0 \text{ for large R and small } \omega L$$
$$= 10^0 \sim 15^0 \text{ for R} = \omega L$$
$$= 15^0 \sim 20^0 \text{ for small R and large } \omega L$$

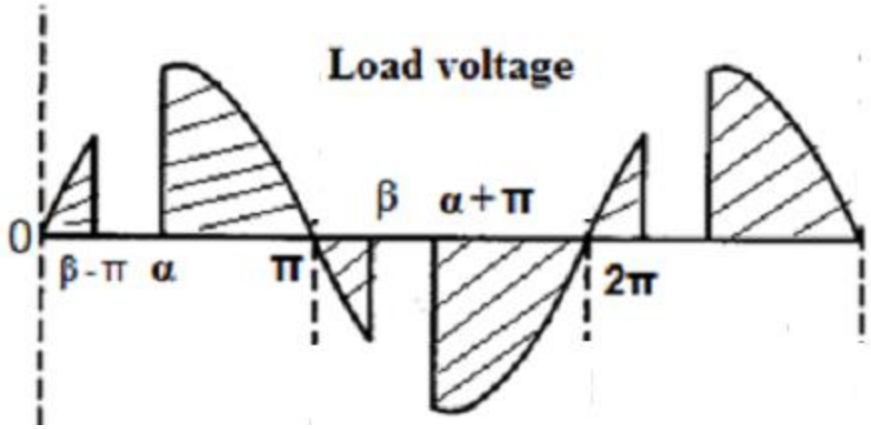
For the previous example,

$$\theta = 60^0, \Delta = 15^0 - 20^0$$
$$\therefore \beta = 180^0 + 60^0 - 15^0 = 225^0$$
$$or$$
$$\beta = 180^0 + 60^0 - 20^0 = 220^0$$

$$\beta = \frac{225+220}{2} = 222.5^0$$

Load voltage waveform analysis

The load voltage waveform for R-L load with AC chopper is shown in Fig.



This waveform can be represented as :

$$v_L(\omega t) = V_m \sin \omega t \Big|_{0,\alpha,\pi+\alpha}^{\beta-\pi,\ \beta,\ 2\pi}$$

= 0 elsewhere

Fourier analysis of the above waveform,

$$a_o = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\, d\omega t = 0$$

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\, cos\omega t\, d\omega t$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} V_m sin\omega t\ \ cos\omega t\, d\omega t$$

$$a_1 = \frac{1}{\pi}\int_0^{\beta-\pi} V_m sin\omega t\ cos\omega t\, d\omega t + \frac{1}{\pi}\int_\alpha^{\beta} V_m sin\omega t\ cos\omega t\, d\omega t + \frac{1}{\pi}\int_{\alpha+\pi}^{2\pi} V_m sin\omega t\ cos\omega t\, d\omega t$$

Use the trigonometric relation sin2x =2 sinx cosx

$$a_1 = \frac{V_m}{2\pi}[\int_0^{\beta-\pi} sin2\omega t\ d\omega t + \int_\alpha^{\beta} sin2\omega t\ d\omega t + \int_{\alpha+\pi}^{2\pi} sin\omega t\ d\omega t]$$

$$a_1 = \frac{V_m}{4\pi}\left[-cos2\omega t\left\{\begin{matrix}\beta-\pi\\0\end{matrix} - cos2\omega t\left\{\begin{matrix}\beta\\\alpha\end{matrix} - cos2\omega t\left\{\begin{matrix}2\pi\\\pi+\alpha\end{matrix}\right.\right.\right.\right]$$

$$a_1 = \frac{V_m}{2\pi}[cos2\alpha - \cos 2\beta]$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} v_L(\omega t)\, sin\omega t\, d\omega t$$

$$b_1 = \frac{1}{\pi}\int_0^{\beta-\pi} V_m sin\omega t\ sin\omega t\, d\omega t + \frac{1}{\pi}\int_\alpha^{\beta} V_m sin\omega t\ sin\omega t\, d\omega t + \frac{1}{\pi}\int_{\alpha+\pi}^{2\pi} V_m sin\omega t\ sin\,\omega t\, d\omega t$$

$$b_1 = \frac{V_m}{2\pi}[2(\beta-\alpha) - \sin 2\beta + sin2\alpha]$$

Worked Examples

Example 1: An ideal single-phase voltage source $V_s = V_m \sin \omega t$ supplies to a series R-L load of phase-angle $\theta = 60^\circ$ via a pair of inverse-parallel connected SCRs. The SCR firing angle $\alpha = 120^\circ$. Calculate the current extinction angle β by an approximate method and use iteration to obtain the accurate value.

Solution

(1) By approximate method $\beta \cong \pi + \theta - \Delta$, For $\theta = 60^\circ$, $\Delta \simeq 15^\circ$.

$\therefore \beta = 180 + 60 - 15 = 220^\circ$.

(2) By the accurate method:

$$\sin(\beta - \theta) = \sin(\alpha - \theta)\, e^{(R/L)\left(\frac{\alpha - \beta}{\omega}\right)}$$

or

$$\sin(\beta - \theta) = \sin(\alpha - \theta)\, e^{-\cot\theta\,(\beta - \alpha)}$$

where: $\frac{R}{\omega L} = \cot\theta = \cot 60^\circ = \frac{1}{\tan\theta} = 0.58$.

$\alpha = 120^\circ = \frac{2\pi}{3} = 2.09$ rad.

$\sin(\alpha - \theta) = \sin(120 - 60) = \sin 60 = 0.866$.

Iterative Evaluation of β.

β		$\sin(\beta - 60^\circ)$	$\sin(\alpha-\theta)\, e^{-\cot\theta(\beta-\alpha)}$
degrees	rad	(LHS)	
220	3.84	0.342	0.314
222	3.87	0.309	0.308

Example 2:

Two SCRs are connected in inverse-parallel for the voltage control of a single-phase series R-L load of phase angle $\theta = 45^\circ$. These are each triggered at a firing-angle $\alpha = 90^\circ$ with reference to thier respective anode voltage waveforms. Estimate (do not calculate) the current extinction angle and use this to calculate the load voltage displacement factor.

Solution:

The current extinction angle β is:

$$\beta = \pi + \theta - \Delta$$

with $\theta = 45^\circ$, $\Delta \cong 10^\circ$

$$\therefore \beta = \pi + 45 - 10 = 180 + 45 - 10 = 215^\circ \text{ or } 3.75 \text{ rad.}$$

$$a_1 = \frac{V_m}{2\pi}(\cos 2\alpha - \cos 2\beta)$$

$$= \frac{V_m}{2\pi}(-1 - 0.342) = \frac{-V_m}{2\pi}(1.342)$$

$$b_1 = \frac{V_m}{2\pi}\left[2(\beta - \alpha) - \sin 2\beta + \sin 2\alpha\right]$$

$$= \frac{V_m}{2\pi}\left[2(3.75 - 1.57) - 0.94 + 0\right]$$

$$= \frac{V_m}{2\pi}(3.42).$$

$$\psi_1 = \tan^{-1}\frac{a_1}{b_1} = \tan^{-1}(-0.392) = -21.4^\circ.$$

Displacement Factor $= \cos\psi_1 = \cos 21.4^\circ = 0.93$.

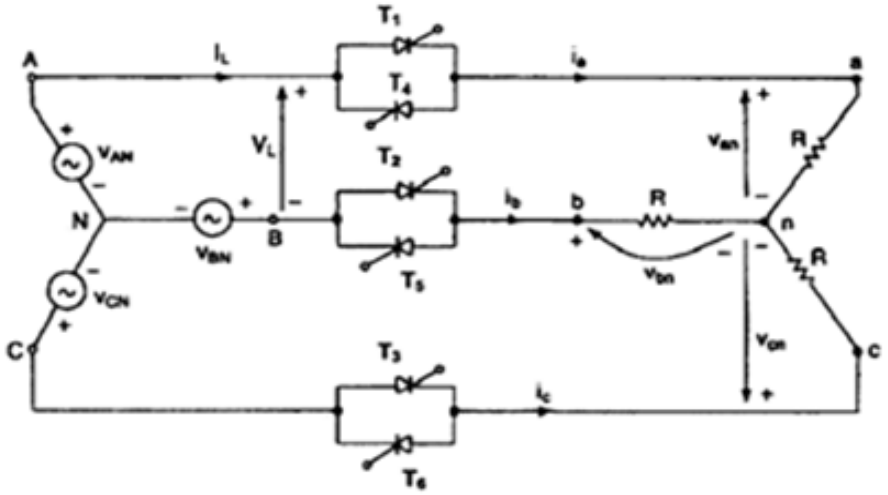
12. Three-phase AC Controllers

Fully controlled Three- phase AC controller

A 3-phase full –wave fully controlled AC controller is shown in Fig.1. With a star connected resistive load.

The firing sequence of thyristors is T1,T6,T2,T4,T3,T5

Fig.1



- For current to flow it is necessary to trigger at least two thyristors at a time.
- If we define the instantaneous input phase voltages as:

  $v_{an} = V_m \sin \omega t$

  $v_{bn} = V_m \sin (\omega t - 2\pi / 3)$

  $v_{cn} = V_m \sin (\omega t - 4\pi / 3)$

- The instantaneous input line voltages are

  $v_{ab} = \sqrt{3}\, v_{an} = \sqrt{3}\, v_m \sin (\omega t + \pi/6)$

  $v_{bc} = \sqrt{3}\, v_{bn} = \sqrt{3}\, v_m \sin (\omega t - \pi/2)$

  $v_{ca} = \sqrt{3}\, v_{cn} = \sqrt{3}\, v_m \sin (\omega t - 7\pi/6)$

The waveforms the input voltages, conduction angles of thyristors and output phase voltages are shown in Fig.2 for $\alpha = 30^{\circ}$

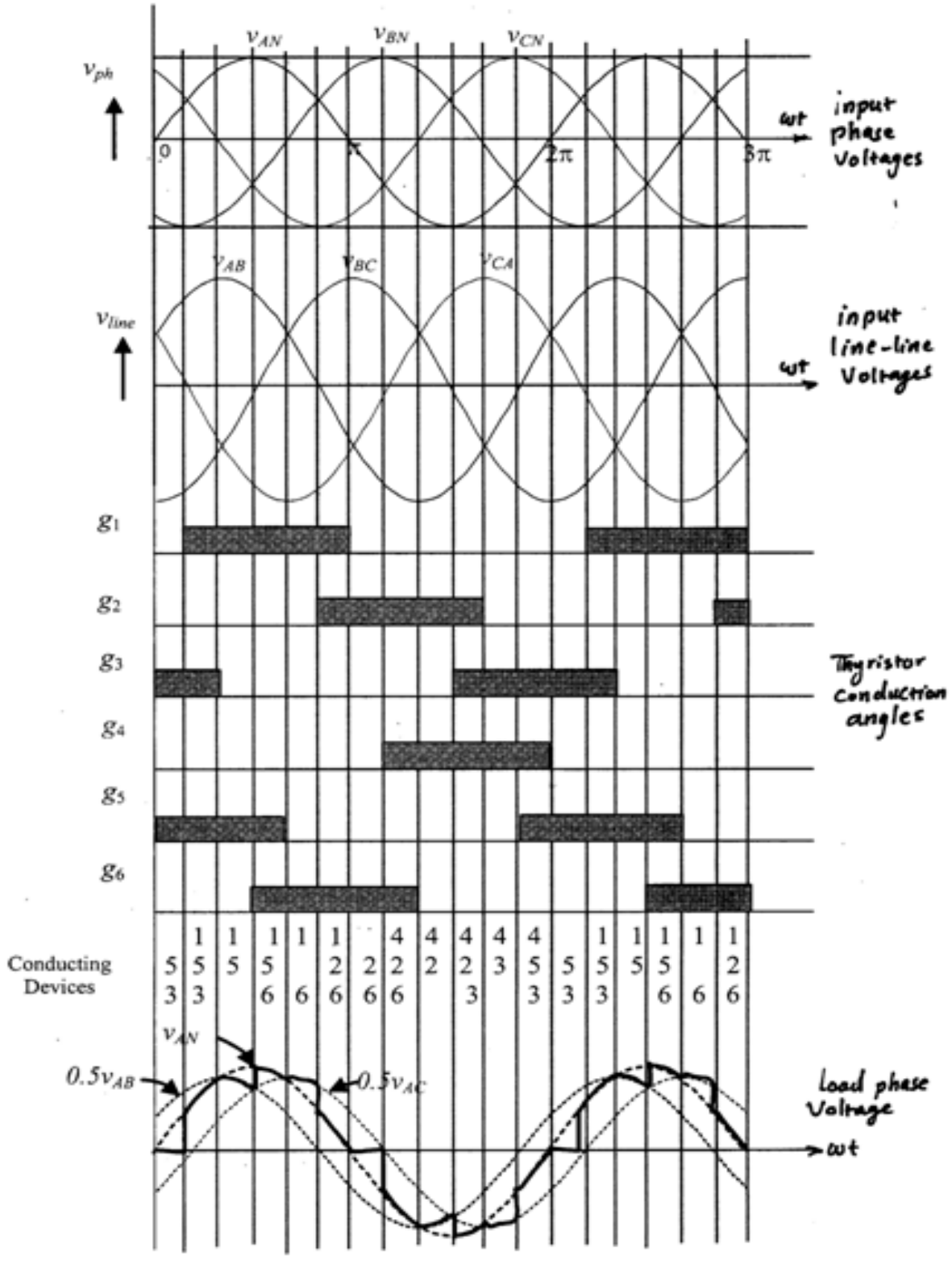


Fig.2 Waveforms

For complete control of the thyristors using the triggering pattern shown each thyristor must be triggered by two separated by 60 degree in time.

Hence we have generally three distinct ranges of firing angle:

1- Range (1)	$0^\circ \leq \alpha < 60^\circ$
2- Range (2)	$60^\circ \leq \alpha < 90^\circ$
3- Range (3)	$90^\circ \leq \alpha \leq 150^\circ$

Similar to half wave controllers the expression for the R.M.S output phase voltage depends on the range of firing angles.

The R.M.S output voltage for a Y– connection loads are found to be:

1- For $0^\circ \leq \alpha < 60^\circ$

$$V_o = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

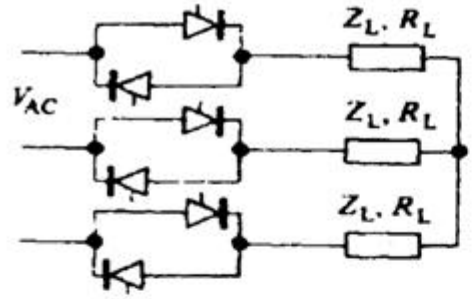
2- For $60^\circ \leq \alpha < 90^\circ$

$$V_o = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{\pi}{12} - \frac{3\sin 2\alpha}{6} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

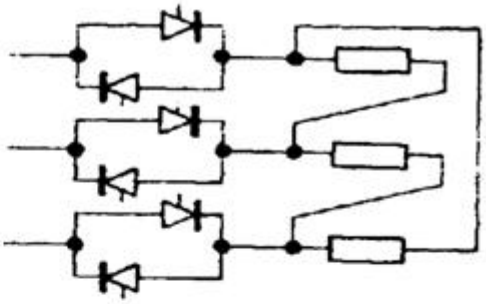
3- For $90^\circ \leq \alpha \leq 150^\circ$

$$V_o = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{5\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

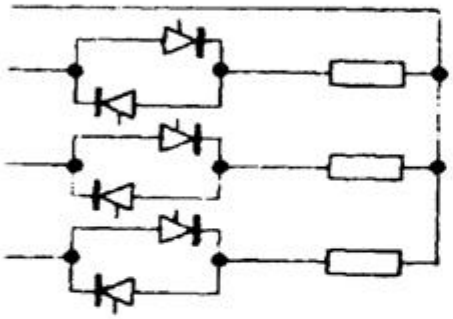
Other types of 3-Phase AC controllers



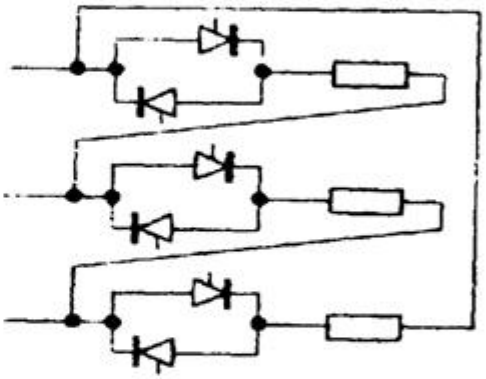
A 3-wire star load



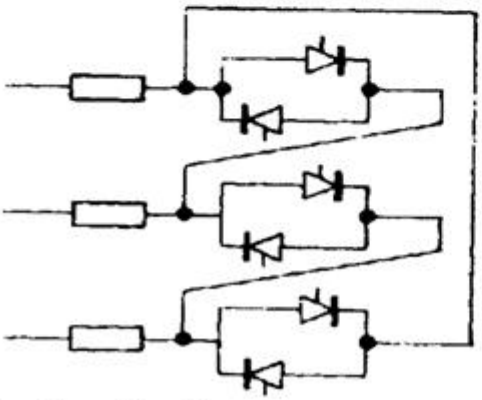
B 3-wire delta load



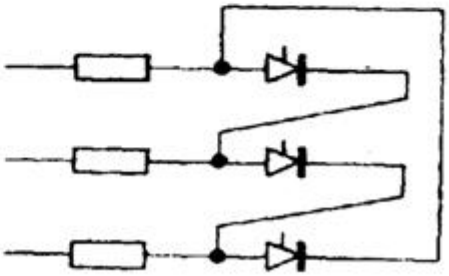
C 4-wire star load



D 3-wire delta controller



E Control in delta



F Half-wave delta control

Applications:

- Phase controlled systems are used for controlling the speed of induction motor is shown in Fig.4:

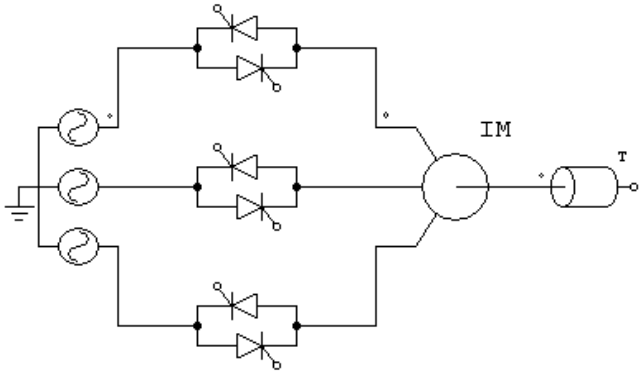


Fig.4 (a) Open loop Speed control of induction Motor using 3-phase AC voltage controller

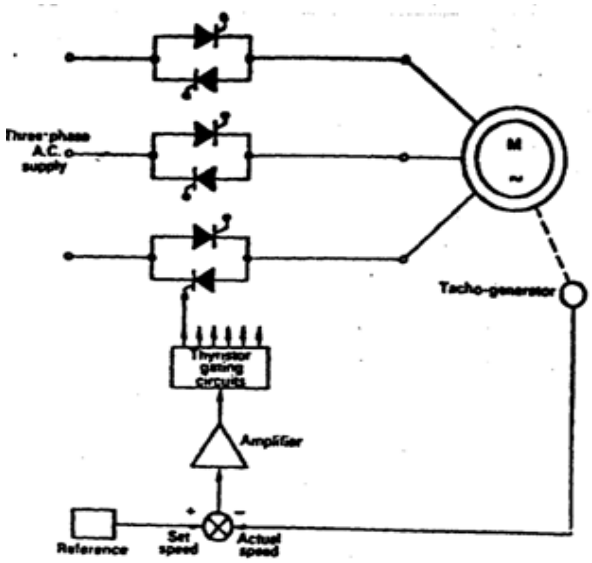


Fig.4(b) Closed loop system for inductive motor speed control using 3-phase AC voltage controller

- If a reversible speed control system is used the line must be change cross connected system Fig.5:

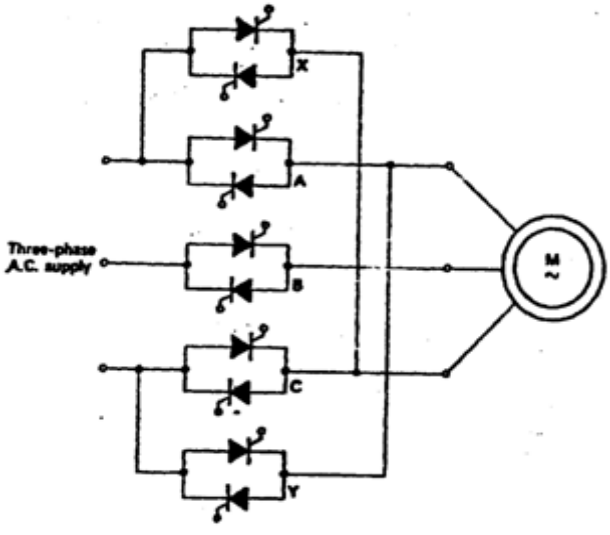


Fig.5 Thyristor configuration of a reversible speed induction drive.

Example:

The three phase bidirectional controller in Fig.6 supplies a Y- connected resistive load of R = 10 Ω and line to line voltage is 208V RMS and *f* =60 Hz. For the delay α=π/6 determine

(a)The output power

(b)The input power factor PF

(c) Give one application of such a converter

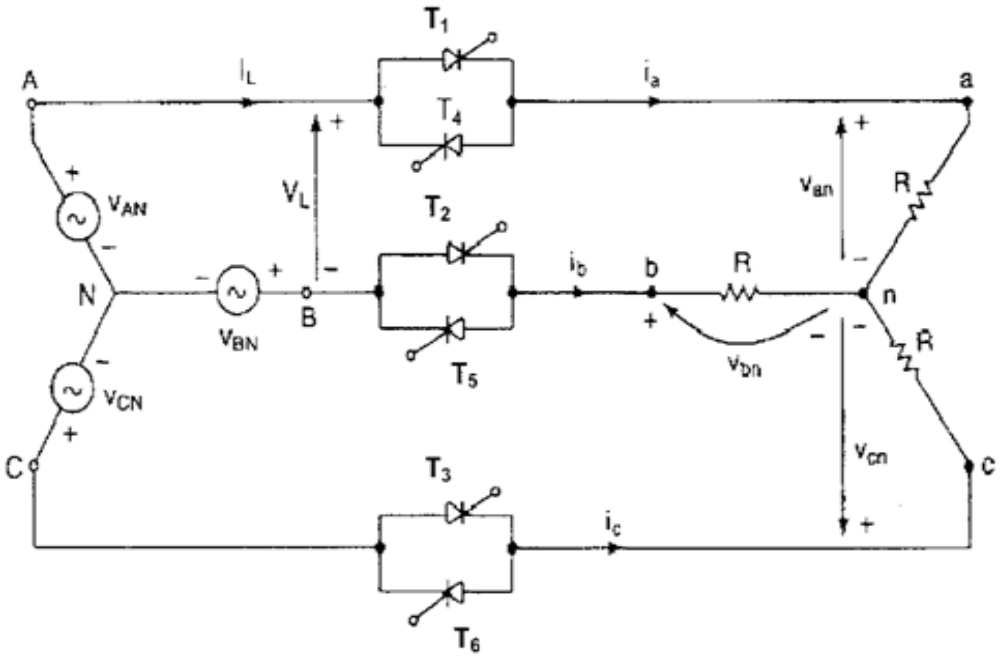


Fig.6

>>> NOT THAT

For $0^\circ \le \alpha < 60^\circ$

$$Vo = \sqrt{3}\, Vm \sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)}$$

For $60^\circ \le \alpha < 90^\circ$

$$Vo = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{\pi}{12} - \frac{3\sin 2\alpha}{6} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

For $90^\circ \leq \alpha \leq 150^\circ$

$$Vo = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{5\pi}{24} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{16} + \frac{\sqrt{3}\cos 2\alpha}{16}\right)}$$

Solution:

(a) For $0^\circ \leq \alpha < , 60^\circ$

$$V_o = \sqrt{3}\, V_m \sqrt{\frac{1}{\pi}\left(\frac{\pi}{6} - \frac{\alpha}{4} + \frac{\sin 2\alpha}{8}\right)} = 117.38V$$

$$I_o = \frac{Vo}{R} = \frac{117.38}{10} = 11.74A$$

$$P_o = 3I_o{}^2 R = 3*(11.74)^2 *10 = 4134W$$

(b) Since the load is Y-connection the phase current. Thus the input volt-ampere power is

$$S = 3V_s I_o = 3*120*11.74 = 4226VA$$

$$PF = \frac{P_o}{s} = \frac{4134}{4226} = 0.978lagging$$

(c) One application of three-phase AC regulator is in electric oven control or induction motor speed control.

13. DC – to – AC Converters

Inverters

Inverter is a device which converts DC voltages (or current) to AC voltages (or current).Inverter converting voltage is called VOLTAGE SOURCE INVERTER (VSI), while inverter converting current is called CURRENT SOURCE INVERTER (CSI). The output of the inverter could be at any desired frequency, voltage or current.

Power transistor such as BJT, MOSFET and IJBT, are widely used in low and medium power inverters. Thyristors or SCRs inverters are used for high power inverter.

So there are many types of inverters available in industry today, and each of them is designed to suit particular application or to meet designed performance requirements. Generally, inverters are categorized into two mains types of voltage source inverters (VSI) and current source inverter (CSI).

For VSI inverter, there are the following common types used in industry:

1- Parallel inverter.

2- Single phase inverter.

-Single-phase half-Bridge inverter.

-Single-phase full-Bridge inverter.

3-Three-phase inverters.

13.1 Parallel Inverter

The Parallel Inverter and Parallel capacitor inverter:

13.1.1 Transistor parallel inverter

The simplest type of voltage source inverter used in low power applications and UPS system for computers is the parallel inverter shown in Figure 13.1.

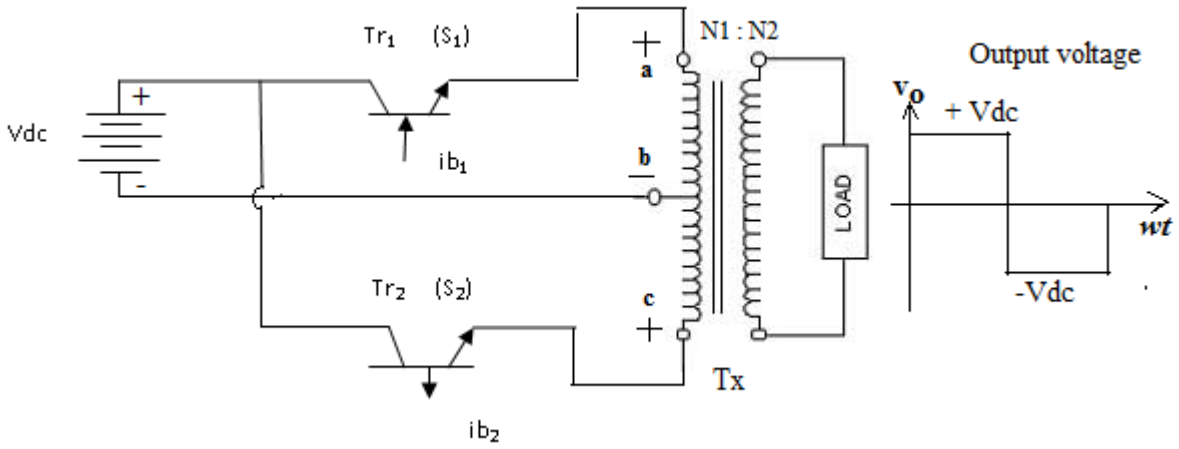
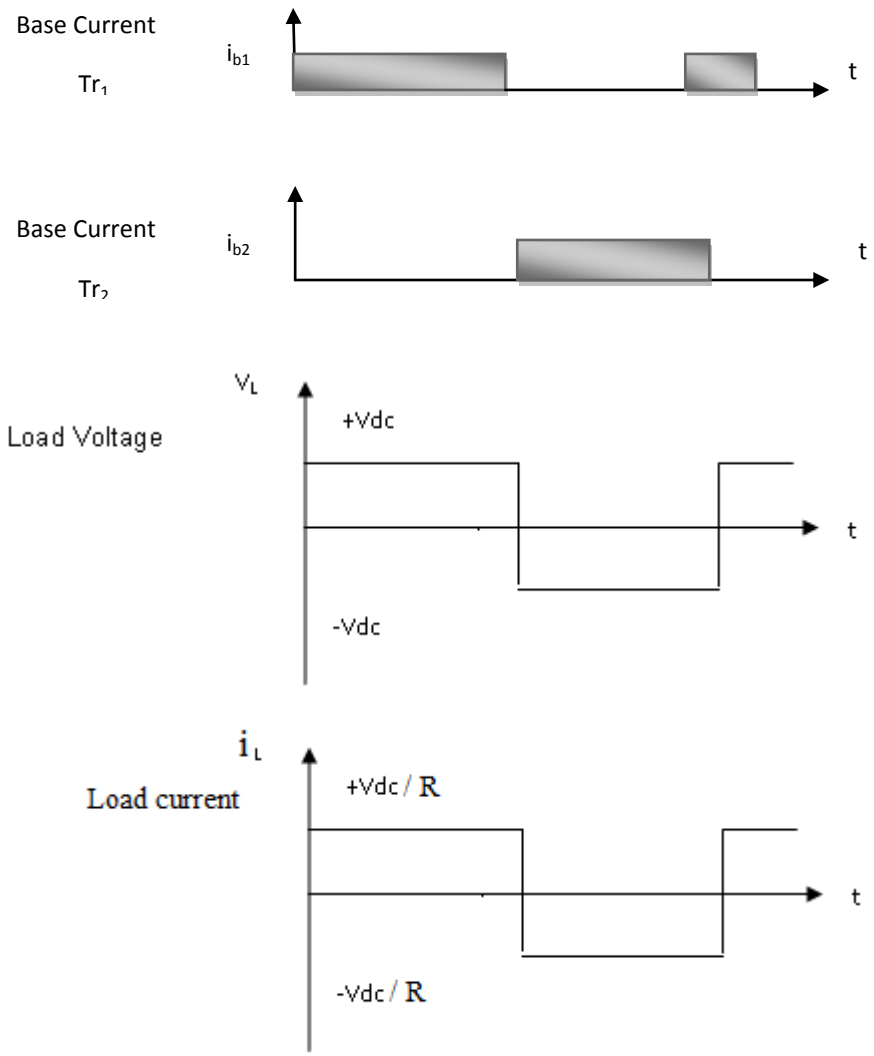


Fig.13.1 Transistor parallel inverter circuit.

The basic single- phase parallel inverter circuit consists of two transistors Tr_1 and Tr_2 that are alternatively turned ON and OFF.

Assuming Tr_1 conducts and Tr_2 OFF, Tr_1 current flows through the primary winding (a-b) of the output transformer Tx and produces an induced voltage on the secondary winding. Hence, supplies a load with positive voltage + Vdc.

When Tr_1 switched off (ib1→ o) and Tr_2 is switched on by applying current ib_2 in its base, current flows through the primary winding (c-b) of the output transformer Tx and produce – V_{dc} at the secondary of Tx . Hence a square wave of amplitude V_{dc} is generated. This square or rectangular wave can be approximated to sine wave by using additional filter. Waveforms of this inverter are shown in Fig.13.2 for pure resistive load.



Conducting Device Sequence	Tr_1	Tr_2
	Upper	Lower

Fig.13.2 Output voltage and current waveforms for the parallel transistor inverter.

13.1.2 Thyristor parallel inverter

The **basic single- phase parallel inverter circuit** consists of two SCRs : SCR 1 and SCR 2, an inductor L, an output transformer, and commutating capacitor C as depicted in Fig.13.3 . The output voltage and current are V_o and I_o respectively. The function of L is to make the source current constant. During the working of this inverter, capacitor C comes in **parallel** with the load via the

transformer. So it is called a **parallel capacitor inverter**. The operation of this inverter can be explained in the following modes.

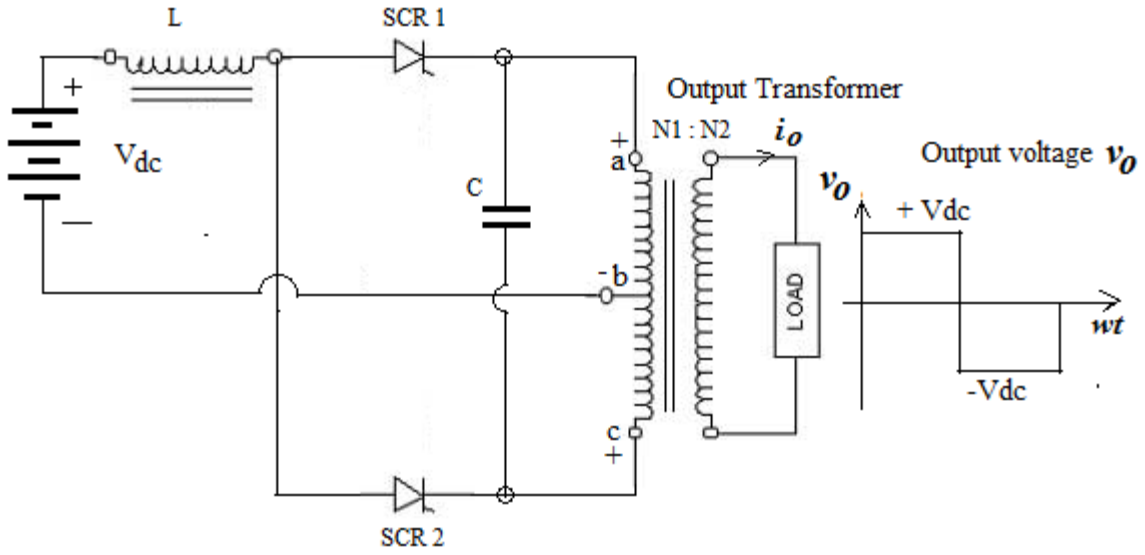


Fig.13.3 Thyristor parallel capacitor inverter circuit.

Mode I

In this mode, SCR1 is conducting while SCR 2 is OFF. In this case current flows in the upper half of the primary winding (section a-b) of the output transformer. As a result an induced voltage on the secondary is produced and supplied to the load. At this time the voltage between a and c terminals, V_{ac}, is equal two times the dc supply voltage .In other words total voltage across primary winding is 2 V_{dc} . Now the capacitor C charges to a voltage of $2V_{dc}$ with upper plate as positive.

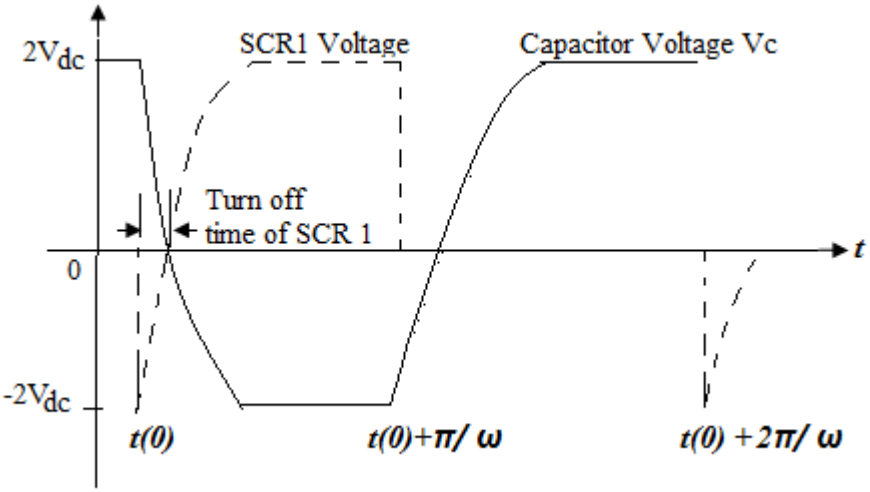
Mode II

When SCR 2 is turned ON by applying a trigger pulse to its gate m at this time (t=0), capacitor voltage 2Vdc appears as a reverse bias across SCR 1, it is therefore turned OFF. A current I_o begins to flow through SCR 2 and lower half of primary winding. Now the capacitor has charged (upper plate as negative) from $+2V_{dc}$ to $-2V_{dc}$ at time $t=t_1$. Load voltage also changes from V_{dc} at t=0 to $-V_{dc}$ at $t=t_1$.

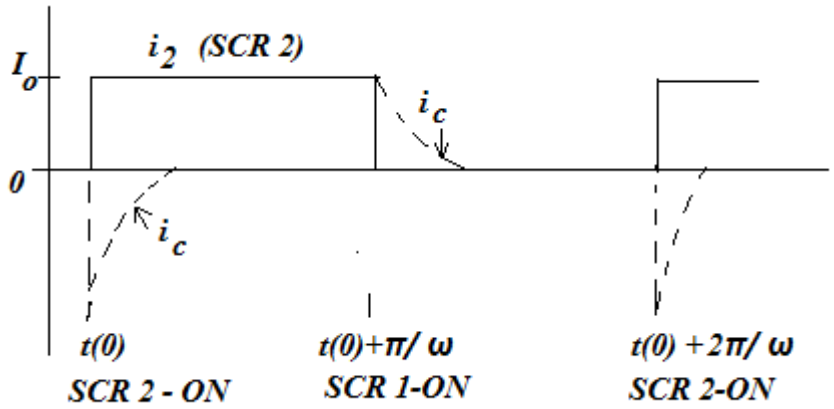
Mode III

When capacitor has charged to $-V_{dc}$, SCR 1 may be tuned ON at any time. When SCR 1 is triggered, capacitor voltage $2V_{dc}$ applies a reverse bias across SCR 2, it is therefore turned OFF. After SCR 2 is OFF, capacitor starts discharging, and charged to the opposite direction, the upper plate as positive. These procedures (Modes) proceed and repeat.

The inductor L in Fig.13.1 is used also to limit the commutating current during commutation process. In the absence of L, The capacitor will charged and discharged very rapidly during each of SCR s' conduction. The result is that the SCR may have no time to turn off. The waveforms of the SCR and capacitor voltages and currents are shown in Fig.13.4 (a) and (b) respectively.



(b)



(b)

Fig.13.4 Waveforms of the SCR and capacitor (a) voltage, and (b) current for the parallel inverter.

13.2 Harmonic Analysis of the Output Voltage Waveform:

The output voltage waveform of the inverter is a square wave (fig.13.5) which is non-sinusoidal. Since this wave is periodic, its harmonic content can be found using Fourier series as follows:

The Fourier coefficients are,

$$\frac{a_o}{2} = \frac{1}{2\pi}\int_0^{2\pi} f(\omega t)d\omega t$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)cosnwt\ d\omega t$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\omega t)sinnwt\ d\omega t$$

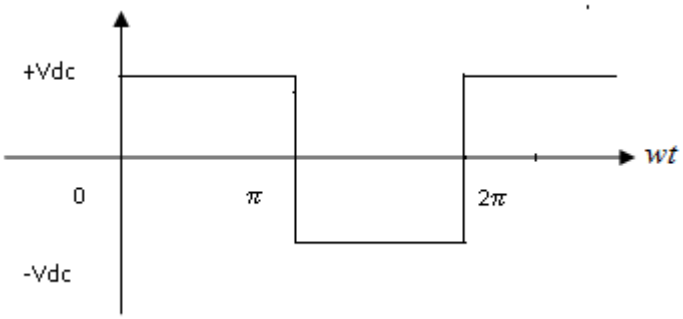


Fig.13.5

The square wave is defined as,

$$f(\omega t) = v_L(\omega t) = Vdc \qquad 0 \le \omega t \le \pi$$

$$= -Vdc \qquad \pi \le \omega t \le 2\pi$$

The dc component is:

$$\frac{a_o}{2} = \frac{1}{2\pi}\left[\int_0^{\pi} Vdc\ d\omega t + \int_{\pi}^{2\pi}(-Vdc)d\omega t\right] = 0$$

$$a_n = \frac{Vdc}{\pi}\left[\int_0^{\pi} cosn\omega t\ d\omega t - \int_{\pi}^{2\pi} cosn\omega t\ d\omega t\right] = 0$$

$$b_n = \frac{Vdc}{\pi}\left[\int_0^{\pi} sinn\omega t\ d\omega t - \int_{\pi}^{2\pi} sinn\omega t\ d\omega t\right]$$

$$= \frac{Vdc}{n\pi}\left[-\ cosn\omega t\ \Big|_0^{\pi} + \ cosn\omega t\ \Big|_{\pi}^{2\pi}\right]$$

$$= \frac{Vdc}{n\pi}\left[(coso - cosn\pi) + (1 - \ cosn\pi\ )\right]$$

$= \frac{Vdc}{n\pi}[1 - cosn\pi + 1 - cosn\pi] = \frac{2Vdc}{n\pi}(1 - cosn\pi)$

- When n is even (2, 4, 6…) cosnπ = 1, $\therefore b_n = 0$.
- When n is odd (3, 5, 7…) cosnπ = -1

$\therefore b_n = \frac{4Vdc}{n\pi}$

The Fourier series is given by:

$f(\omega t) = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n\, cosn\omega t + b_n sinn\omega t)$

Hence the output (load voltage) can be represented by Fourier series as,

$$\therefore V_L(\omega t) = 0 + \sum_{n=1}^{\infty}(0 + \frac{4Vdc}{n\pi} sinn\omega t)$$

or $V_L(\omega t) = \sum_{n=1}^{\infty} \frac{4Vdc}{n\pi} sinn\omega t$

The fundamental component (n=1) is,

$V_{L1}(\omega t) = C_1 = \frac{4Vdc}{\pi} sin\omega t$ (peak value of the first harmonic)

The R. M. S value of the fundamental component,

$V_{L1}(rms) = \frac{C_1}{\sqrt{2}} = \frac{\frac{4Vdc}{\pi}}{\sqrt{2}} = \frac{4Vdc}{\sqrt{2}\pi}$

Similarly, the peak amplitude of 3rd harmonic n=1 can be evaluated as,

$V_{L3}(\omega t) = \frac{4Vdc}{3\pi} sin3\omega t$

And the R.M.S value of the 3rd harmonic is,

$$V_{L3}(rms) = \frac{4Vdc}{3\sqrt{2}\pi}$$

And so on, the R.M.S. values for the 5^{th}, 7^{th}, …. odd harmonics can be evaluate.

Hence, the harmonic spectrum of the square wave is as shown in Fig.13.6.It is clear from this figure that:

- Harmonic amplitudes decrease as n increases.
- No even harmonics.
- Nearest harmonic is the third, if the fundamental is 50 Hz, then the third harmonic is 150Hz and the fifth harmonic 250 Hz as shown in Fig.13.7.

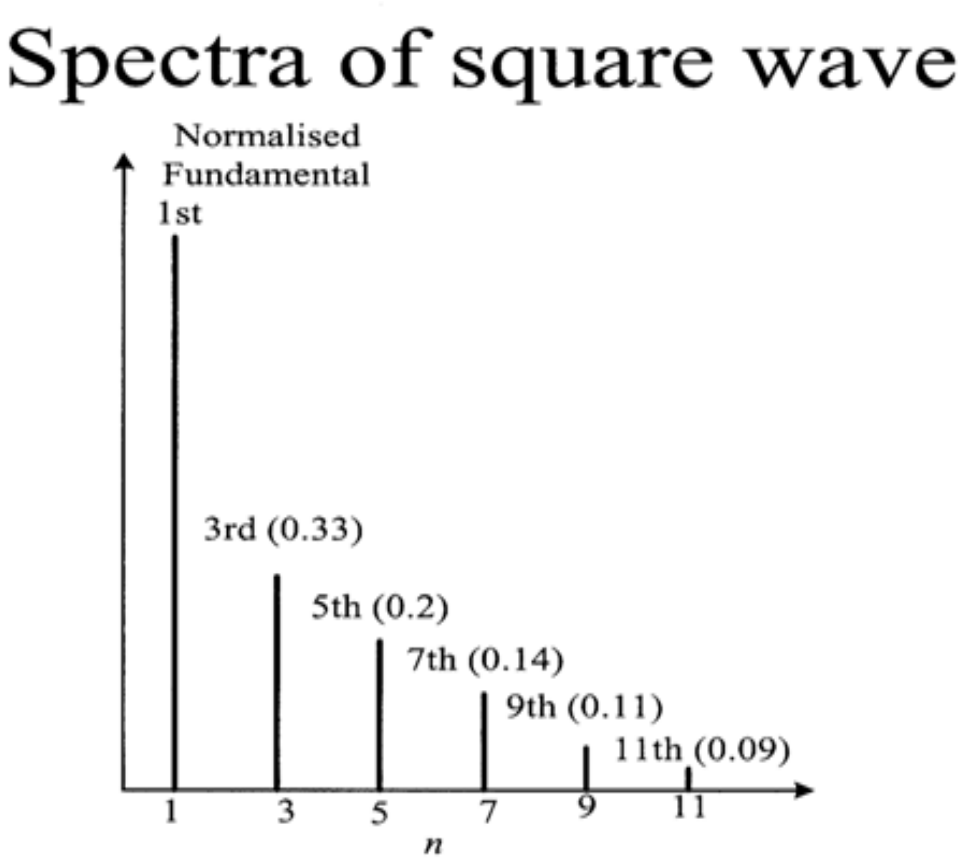


Fig.13.6 Harmonic spectrum of the square wave.

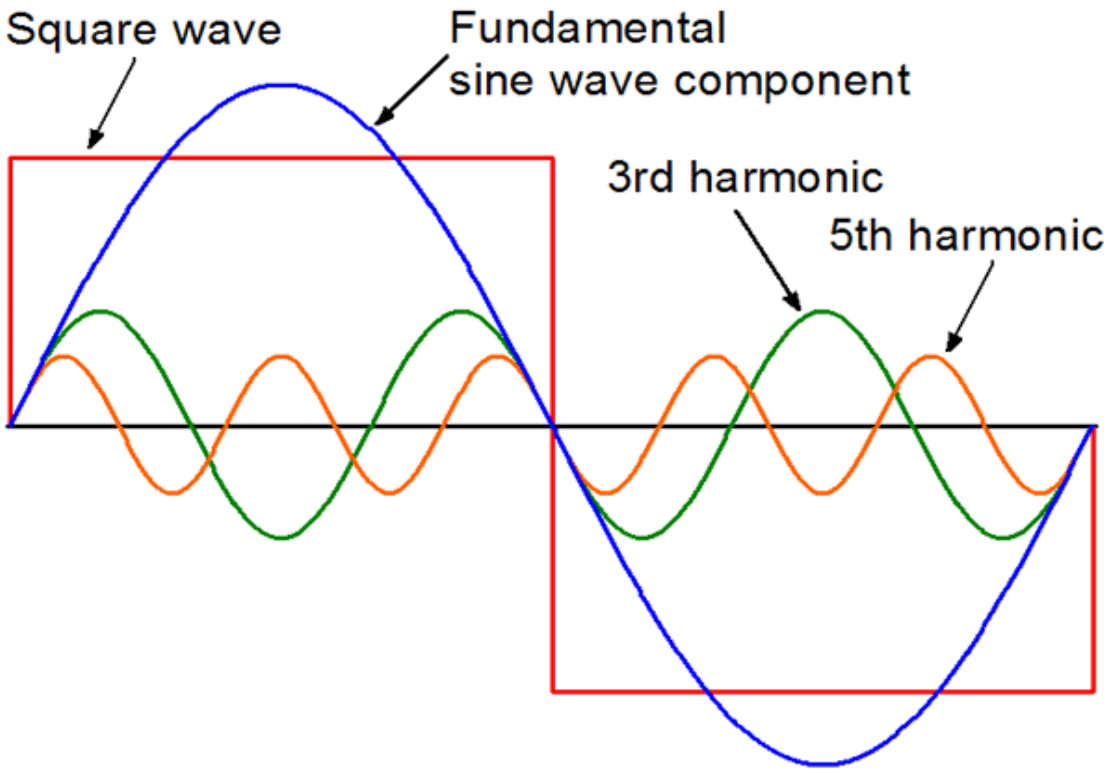


Fig.13.7 The square wave and its odd harmonics.

Example: A parallel inverter has a supply dc voltage of 100V. Supply a resistive load $R = 10\,\Omega$ via a center-tap transformer with 1:1 ratio. The output frequency is 50Hz. Determine:

(a) The rms value of the output voltage waveform

(b) The amplitude of the Fourier series terms for the square output voltage waveform up to 9th order harmonics.

(c) The rms value of the output voltage in terms of harmonics, compare it with that obtained in (a).

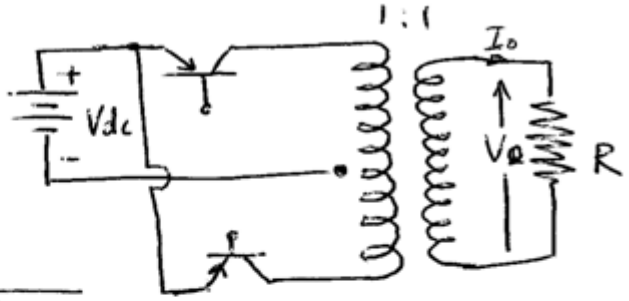
(d) The power absorbed by the load (consider up to 9th order harmonic.

(e) Draw the frequency spectra of the output voltage waveform.

(f) Calculate the total harmonic distortion factor (THD)

Solution:

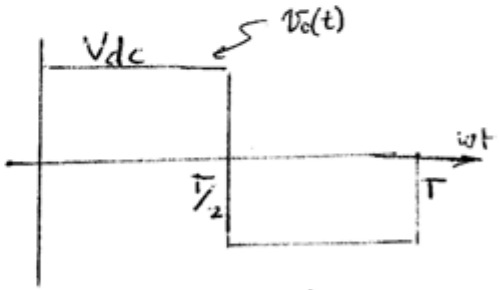
(a) The rms value of the output voltage is found as follows:



$$V_{orms} = \sqrt{\frac{1}{T/2}\left[\int_0^{T/2} V_{dc}^2\, dt\right]}$$

$$= V_{dc} = 100V$$

$$\therefore V_{orms} = 100V$$



output voltage waveform

Ex/1

(b) The Fourier series of the output voltage is:

$$v_o(t) = \sum_{1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{n\pi} \sin n\omega t .$$

The amplitute of the nth harmonic C_n is:

$$C_n = \frac{4V_{dc}}{n\pi} = \frac{4\times100}{n\pi} = \frac{127.3}{n}$$

$$v_o(t) = C_1 \sin\omega t + C_3 \sin 3\omega t + C_5 \sin 5\omega t + C_7 \sin 7\omega t + C_9 \sin 9\omega t$$

$$= \frac{127.3}{1}\sin\omega t + \frac{127.3}{3}\sin 3\omega t + \frac{127.3}{5}\sin 5\omega t + \frac{127.3}{7}\sin 7\omega t + \frac{127.3}{9}\sin 9\omega t .$$

$$\therefore v_o(t) = 127.3\sin\omega t + 42.4\sin 3\omega t + 25.5\sin 3\omega t + 18.2\sin 7\omega t + 14.1\sin 9\omega t .$$

(C) In terms of the harmonics

$$V_{o\,rms}^{h} = \sqrt{\left(\frac{C_1}{\sqrt{2}}\right)^2 + \left(\frac{C_3}{\sqrt{2}}\right)^2 + \left(\frac{C_5}{\sqrt{2}}\right)^2 + \left(\frac{C_7}{\sqrt{2}}\right)^2 + \left(\frac{C_9}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\left(\frac{127.3}{\sqrt{2}}\right)^2 + \left(\frac{42.4}{\sqrt{2}}\right)^2 + \left(\frac{25.5}{\sqrt{2}}\right)^2 + \left(\frac{18.2}{\sqrt{2}}\right)^2 + \left(\frac{14.1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{9592.97} = 97.94\ V$$

$$\therefore V_{o\,rms}^{h} = \underline{\underline{97.94}}$$

This value is less than $V_{o\,rms}$ since we calcalte up to 9th order harmonics only.

EX/2

(d) To calculate the power we must calculate the rms value of the current for each harmonics.

The amplitude of the nth harmonic current:

$$I_n = \frac{C_n}{Z_n}$$

where $Z_n = \sqrt{R^2 + (n\omega L)^2} = R$, since $L = 0$.

$$I_{n_{rms}} = \frac{C_n/\sqrt{2}}{Z_n} = \frac{I_n}{\sqrt{2}}$$

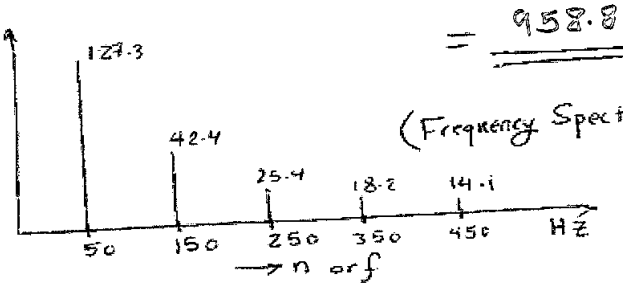
$$\therefore P_n = I_{n_{rms}}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R = \frac{I_n^2}{2} R.$$

n	f_n (Hz)	C_n (V)	Z_n (Ω)	I_n (A)	P_n (W)
1	50	127.3	10	12.73	810.0
3	150	42.4	10	4.24	89.8
5	250	25.5	10	2.55	32.5
7	350	18.2	10	1.82	16.5
9	450	14.1	10	1.41	9.94

The total power is

$$P_T = \sum P_n = P_1 + P_3 + P_5 + P_7 + P_9 = 810 + 89.8 + 32.5 + 16.5 + 9.94 = 958.8 \text{ W}$$

(e)



(Frequency Spectrum)

Ex/3

(f) The total harmonic distortion factor:

$$THD = \sqrt{\frac{V_{o_{rms}}^2 - V_{1_{rms}}^2}{V_{1_{rms}}^2}}$$

$$V_{o_{rms}} = 100$$

$$V_{1_{rms}} = \frac{C_1}{\sqrt{2}} = \frac{127.3}{\sqrt{2}} = 90\ V.$$

$$\therefore THD = \sqrt{\frac{(100)^2 - (90)^2}{(90)^2}} = 0.4843$$

$$\text{or } THD = 0.4843 \times \frac{100}{100} = 48.43\%.$$

This is very high THD. The practical value of THD is (3 - 10)% hence we need to use low pass filter at the output.

14. DC – to – AC Converters

Single-phase inverters:

14.1 Single-phase half-bridge inverter

This type of inverter is very simple in construction. It does not need output transformer like parallel inverter. It sometimes called center-tapped source inverter. The basic configuration of this inverter is shown in Fig 14.1.

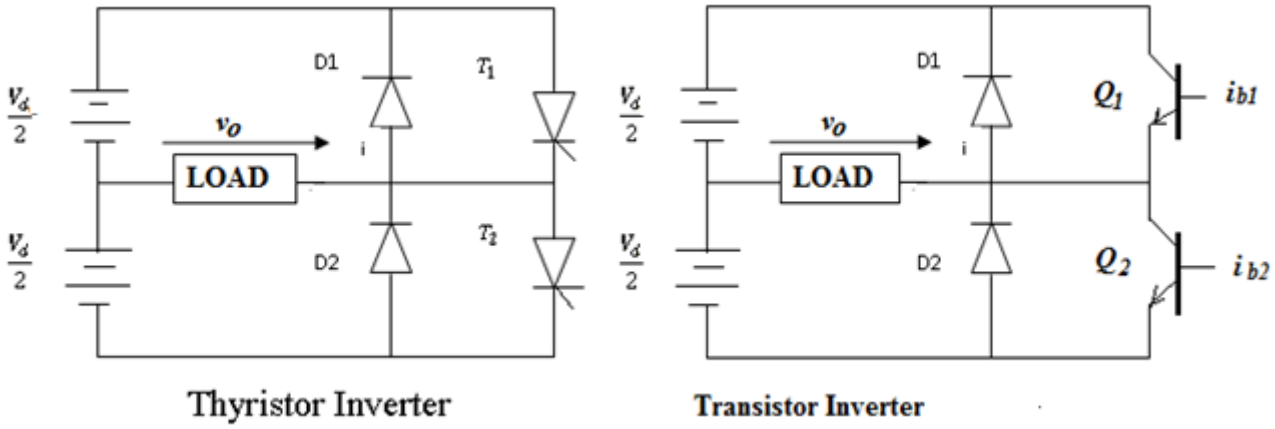
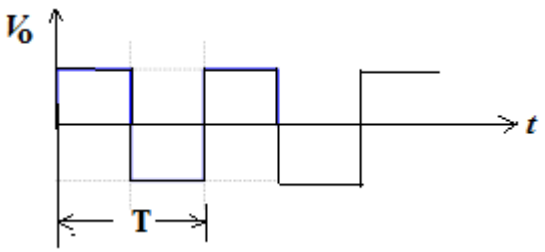


Fig.14.1 Single-phase inverter half-bridge circuit.

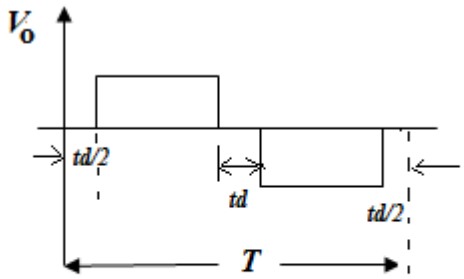
- The top and bottom switch has to be “complementary” i. e. If the top switch is closed (ON), the bottom must be off, and vice-versa.
- The output voltage waveform is a square wave as shown in Fig.14.2.

Fig.14.2 Square wave output Voltage waveform



- In practice, a dead time between ON and OFF (td) for Q_1 & Q_2 is required to avoid short circuit or "shoot-through" faults. This leads to produce quasi-square wave output voltage (Fig.14.2).

Fig.14.3

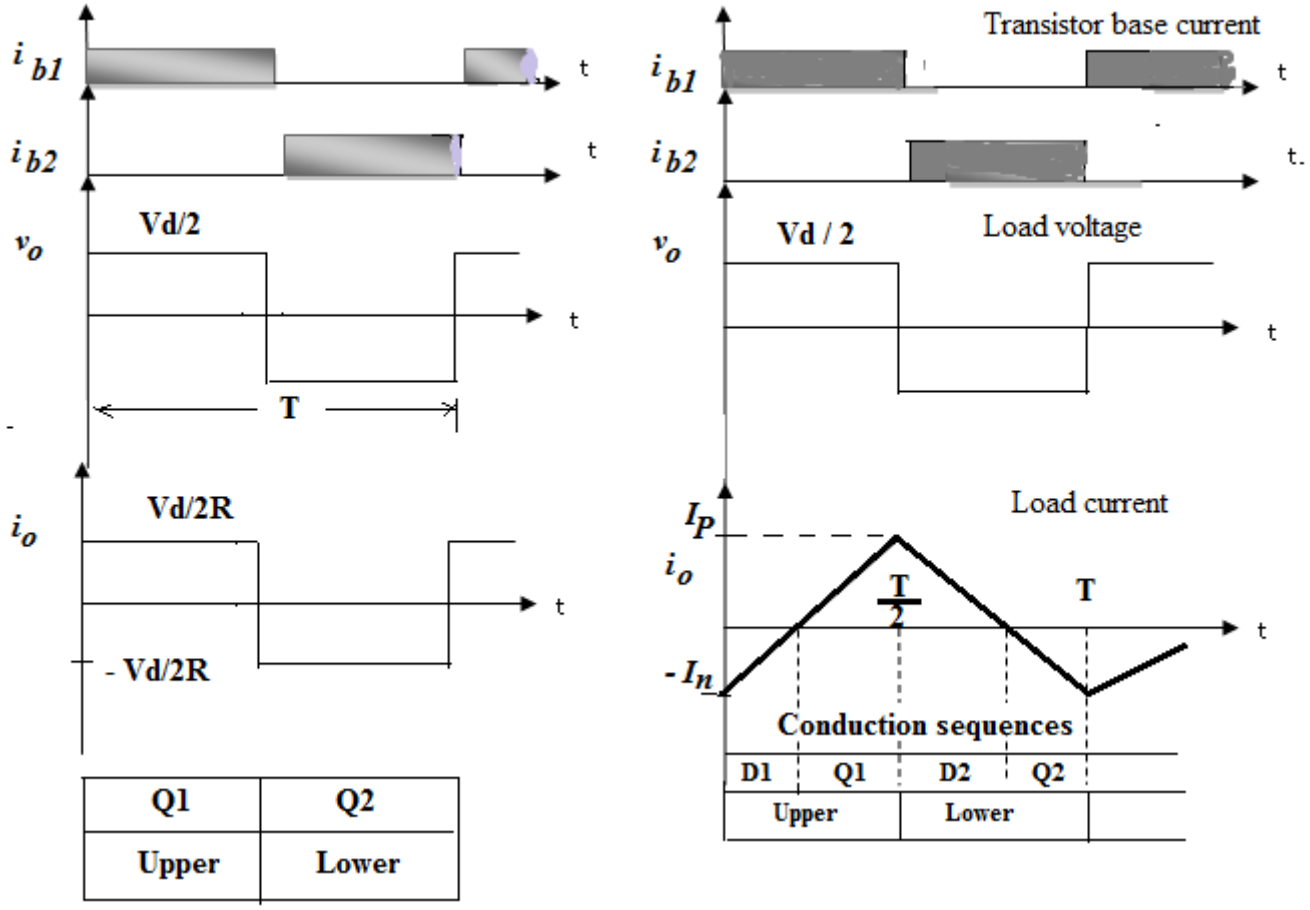


Quasi - square output voltage waveform
***td* = Dead time**

Performance of Half-Bridge Inverter with Resistive - Inductive loads:

- If the load is resistive, the output current waveform will be a copy of the voltage waveform as shown in Fig.14.4 (a). The output voltage is a square (or quasi-square) wave.
- However with an inductive load Fig.14.4 (b), the load current *i* is delayed although the output voltage wave is still a square.

  The current will grow exponentially during the positive half-cycle from $-I_n$ to I_p according to the following equation: $\left(\frac{Vd}{2} = Ri + L\frac{di}{dt}\right)$.
  - Through D1 [load returning power to the upper half of the source].
  - Through Q1 [load absorbing power from the upper half of the source] until $t = \frac{T}{2}$, whereby $i = I_p$.



(a) Operation with Resistive Load (b) Operation with Inductive Load

Fig.14.4 Output voltage and current for single-phase half-bridge

- Negative half cycle starts by conduction of Q2. Current start its change from I_p to zero through D2 and then to $-I_n$ through Q2, according to: $(-\frac{Vd}{2} = Ri + L\frac{di}{dt})$.

  RMS value of the Load voltage:

$$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T V_o^2 dt} = \left[\frac{1}{T}\left(\frac{Vd}{2}\right)^2 T\right]^{\frac{1}{2}} = \frac{Vd}{2}$$

- Load voltage $v_o\ (\omega t)$ can be expressed in terms of harmonics by Fourier series as:

  $v_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi}\left(\frac{Vd}{2}\right) \sin n\ \omega t$, $v_o(\omega t) = 0$ for n=2,4,6…

  Where: $\omega = 2\pi f_0$ is the frequency of the output voltage in (rad/sec) .

- The fundamental component of the load voltage had a peak value $V_{o1P} = C_1 = \frac{4}{\pi}\ \frac{Vd}{2}$, and it has r.m.s value

  $V_{o1rms} = \frac{C_1}{\sqrt{2}} = \frac{\sqrt{2}Vd}{\pi} = 0.45\ Vd = V_{L1}$

- For an R - L load, the instantaneous load current *i* can be found by dividing the instantaneous output voltage by the load impedance Z= R + jnωL , or $Z = \sqrt{R^2 + (n\omega L)^2}$, thus

  $i(\omega t) = \frac{v_o(\omega t)}{Z}$

  $i(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{2Vd}{n\pi\sqrt{R^2+(n\omega L)^2}} \sin\ (n\ \omega t - \Psi n)$

  where $\Psi n = tan^{-1}\frac{n\omega L}{R}$

- If I_{o1} is the r.m.s fundamental load current, the fundamental output power (for n=1) is:

  $P_1 = V_{o1} I_{o1} cos\Psi_1 = I_{o1}^2 R$

  $= \left[\frac{2Vd}{\sqrt{2}\pi\sqrt{R^2+(\omega L)^2}}\right]^2 . R$

Note: In most applications (e. g. electric motor drives) the output power due to the fundamental current is generally the useful power,

and the power due to harmonic currents is dissipated as heat and increases the load temperature.

Example 1: The single-phase half-bridge inverter in Fig. 14.1 has a resistive load of $R = 2.4\ \Omega$ and the d.c. input voltage $V_d = 48\ V$. Determine:

(a) The r.m.s value of the load voltage, V_{orms}.

(b) The r.m.s value of the load voltage at the fundamental frequency V_{L1}.

(c) The output power P_o.

(d) The average and peak currents of each transistor.

(e) The peak reverse blocking voltage V_{BR} of each transistor.

(f) The THD. factor.

Solution:

(a) $V_{orms} = \frac{V_d}{2} = \frac{48}{2} = 24\ V.$

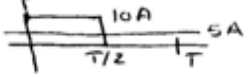
(b) $V_{L1} = 0.45\ V_d = 21.6\ V.$ $\left(V_{L1_{(r.m.s)}} = \frac{\sqrt{2}\,V_d}{\pi}\right)$

(c) $P_o = \frac{V_{or.m.s}^2}{R} = \frac{(24)^2}{2.4} = 240\ W.$

(d) The peak transistor current $I_p = \frac{V_{orms}}{R} = \frac{24}{2.4} = 10A.$

Because each transistor conducts for a 50% duty cycle, the average current of each transistor is

$I_Q = 0.5 \times 10 = 5A.$



(e) The peak reverse blocking voltage $V_{BR} = 2 \times 24 = 48V.$

(f) $THD = \frac{V_h}{V_{L1}} = \frac{\text{Harmonic Voltages}}{\text{fundamental Voltage}}$

$$V_h = \sqrt{\sum_{n=1,3,5}^{\infty} V_{nr.m.s}^2} = \sqrt{V_{Lr.m.s}^2 - V_{L1}^2}$$

$$\therefore THD = \sqrt{\frac{V_{Lr.m.s}^2 - V_{L1}^2}{V_{L1}^2}} = \sqrt{\frac{(24)^2 - (21.6)^2}{(21.6)^2}}$$

$$= 48.34\%.$$

14.2 Single-phase full-bridge inverter

The single-phase bridge – Inverter using BJT transistors is shown in Fig.14.5. This inverter is constructed from two half – bridge inverter using single DC source Vd and the load is connected between the centers of the two legs.

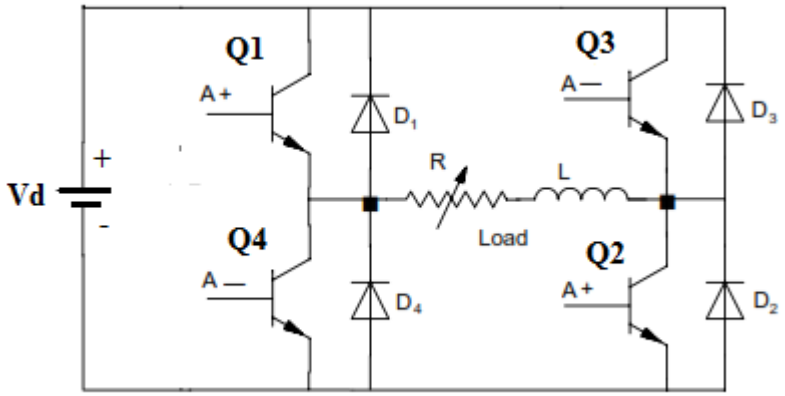


Fig.14.5 Single-phase full-bridge inverter circuit

- **SQUARE-WAVE OUTPUT :**

Q1 and Q2 are triggered simultaneously and so are Q3 and Q4. Each device is made to conduct for half time of the output cycle, the load voltage waveform with the transistor base currents are shown in Fig 14.6.

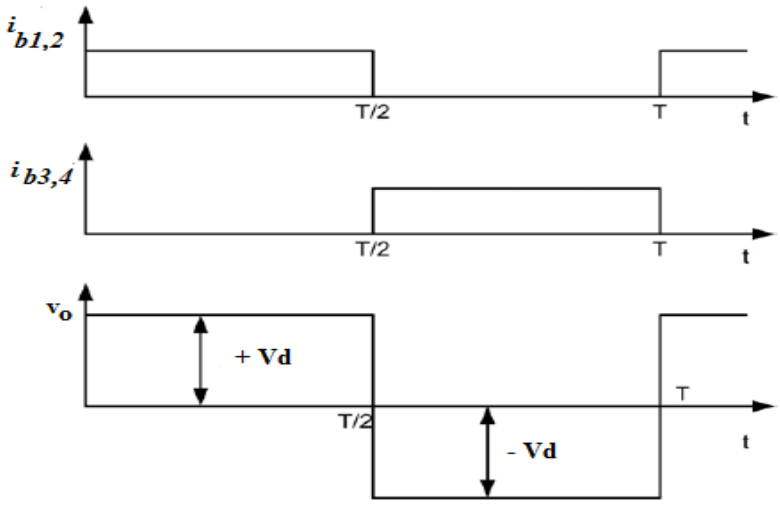


Fig.14.6

Performance with R - L load:

- With resistive - inductive load, the current i_0 lags the square – wave output voltage v_0 as shown in Fig.14.7.

- Triggering Q1 & Q2 connects the load to *Vd*. For steady load condition, i_0 grows exponentially through D1 D2 and then through Q1 Q2 from $-I_n$ to I_p according to $(Vd = Ri + L\frac{di}{dt})$.

- Negative half cycle starts by triggering Q3 & Q4 at $t_1 = \frac{T}{2}$, and Q1 Q2 goes off when $i_{b1} = i_{b2} = 0$ (base current blocking).

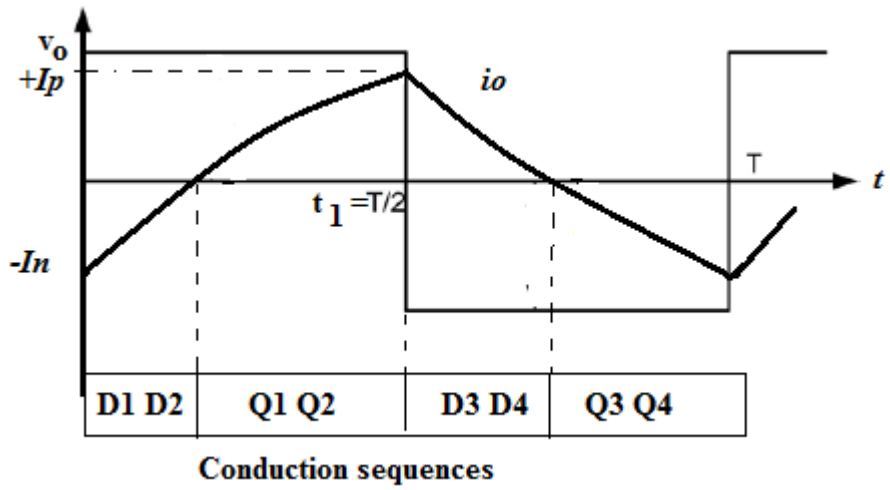


Fig.14.7 current for single-phase full-bridge inverter.

- Load voltage reverses to –Vd and i_o will flow through D3D4 and then through Q3Q4 according to the equation $(-Vd = Ri + L\frac{di}{dt})$. At the end of negative half cycle $i_o = -I_n$.

Note that: $t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega}$

Load r.m.s voltage = $\left[\frac{1}{t_1}\int_0^{t_1} V_d^2\, dt\right]^{\frac{1}{2}} = V_d$.

Load voltage V_0(t) may be expressed as:

$$V_0(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi} Vd \sin n\omega t.$$

The peak value of the fundamental component (n=1) of the load voltage is,

$V_{o1p} = \frac{4Vd}{\pi}$ (maximum or peak value)

The RMS value of the fundamental component is

$V_{o1rms} = \frac{4Vd/\pi}{\sqrt{2}} = \frac{4Vd}{\sqrt{2}\pi} = 0.9\,Vd$ (r.m.s value).

Load r.m.s current and power can be determine from

$$I_{orms} = \left[\frac{2}{T}\int_0^{T/2} i_o^2 dt\right]^{\frac{1}{2}}$$

And P = $\frac{2}{T}\int_0^{T/2} v_o . i_o\, dt.$

where the instantaneous value of the load current i_o for an R-L load is

$$i_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4Vd}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \Psi n)$$

The angle by which the load current lgs the load voltage is.

$$\Psi n = tan^{-1} \frac{n\omega L}{R}.$$

The total harmonic distortion factor is,

$$\text{THD} = \sqrt{\frac{(V_{Lrms}^2 - V_{L1}^2)}{V_{L1}^2}}$$

- When diodes D1 and D2 are conducting, the energy fed back to the source, thus they are known as feedback diodes.

Example: For the single-phase full-bridge, transistor inverter shown in Fig.1, V_{dc}=100V, load resistance R=10 Ω , load inductance L = 25 mH and the output frequency is 60 Hz.

It is required to analyse the circuit by determining: (a) The amplitudes of the Fourier series terms for the output voltage wave up to the 9th order harmonics, (b) The amplitudes of the Fourier series terms for the load current wave up to the 9th order harmonics,(c) The power absorbed by the load in terms of harmonics, (d) compute the total harmonic distortion factor (THD).

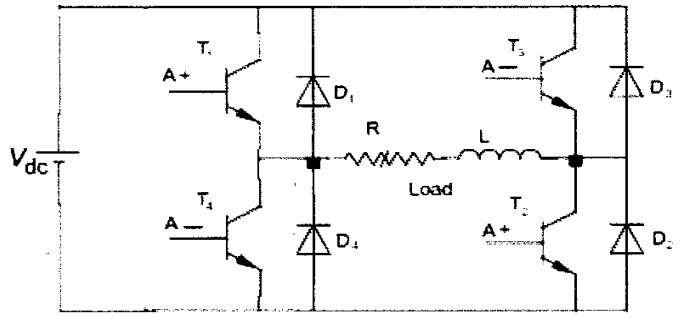


Fig.1

Solution:

(a) The output voltage wave in terms of Fourier series is:

$$v_o(\omega t) = \sum_{n=1,3,5,\ldots}^{n} \frac{4V_{dc}}{n\pi} \sin n\omega t$$

The amplitude C_n of the nth order harmonic is: $C_n = \frac{4V_{dc}}{n\pi} = \frac{4\times 100}{n\pi} = \frac{127.3}{n} = V_n$

$$\therefore v_o(\omega t) = \frac{127.3}{1}\sin\omega t + \frac{127.3}{3}\sin 3\omega t + \frac{127.3}{5}\sin 5\omega t + \frac{127.3}{7}\sin 7\omega t + \frac{127.3}{9}\sin 9\omega t.$$

$$= 127.3\sin\omega t + 42.4\sin 3\omega t + 25.5\sin 5\omega t + 18.2\sin 7\omega t + 14.1\sin 9\omega t.$$

(b) The amplitude of the nth harmonic component of the load current is:

$$I_n = \frac{V_n}{Z_n} \quad \text{where } Z_n = \sqrt{R^2 + (n\omega L)^2}$$

In terms of Fourier series:

$$i_o(\omega t) = \sum_{n=1,3,5,\ldots}^{n} \frac{V_n}{Z_n} \sin(n\omega t - \theta_n)$$

where $\theta_n = \tan^{-1}\frac{n\omega L}{R}$.

(c) $$P_n = I_{n(rms)}^2 \cdot R = \left(\frac{I_n}{\sqrt{2}}\right)^2 \cdot R .$$

n	f_n(Hz)	V_n(V)	$Z_n(\Omega)$	I_n(A)	P_n(W)
1	60	127.3	13.7	9.29	431
3	180	42.4	30.0	1.42	10.0
5	300	25.5	48.2	0.53	1.4
7	420	18.2	66.7	0.27	0.34
9	540	14.1	85.4	0.17	0.14

The total power is $P = \sum P_n = 443$ W.

(d) $$THD = \sqrt{\frac{V_{o,rms}^2 - V_{1,rms}^2}{V_{1,rms}^2}} = \frac{\sqrt{V_{o,rms}^2 - V_{1,rms}^2}}{V_{1,rms}}$$

$V_{o,rms} = V_d = 100$ V.

$V_{1,rms} = \frac{127.3}{\sqrt{2}} = 90$ V

$\therefore THD = \frac{\sqrt{(100)^2 - (90)^2}}{(90)} = \frac{\sqrt{1900}}{90} = 0.4843$

or $0.4843 \times \frac{100}{100} = 48.4\%$.

!4.3 Inverter Output Voltage Control

Many inverter applications require a means of output voltage control. In most of these applications this control is usually required in order to provide stepless adjustment of the inverter output voltage.

The methods of control can be group into three broad categories:-

1. Control of voltage supplies to the inverter
2. Control of voltage delivered by the inverter
3. Control of voltage within the inverter.

- There are a number of well-known methods of controlling the d.c. voltage supplies to an inverter or the a.c. voltage delivered by an inverter. These include the use of d.c. choppers, and phase-controlled rectifiers. The principal disadvantage of these methods is that the power delivered by the inverter is handed twice, once by the d.c. or a.c. voltage control and once by the inverter.
- Control of the inverter output voltage may be achieved by incorporating time-ratio controls within the inverter circuit.
- A method of controlling the voltage within an inverter involves the use of pulse wide modulation techniques. With this technique the inverter output voltage is controlled by varying the duration of the output voltage pulses.

Pulse width voltage and frequency control

A method of controlling the output voltage and frequency within an inverter involves the use of pulse wide modulation techniques. With this technique the inverter output voltage is controlled by varying the duration of the output voltage pulses as shown in Fig.14.8.

PWM is obtained by comparing a reference signal, Ar with a triangular carrier wave, Ac. By varying Ar from 0 to Ac , the pulse width δ can be varied from 0^o to 180^o. The modulation index is defined as,

$$\text{Modulation Index} = M = \frac{Ar}{Ac}$$

The output voltage may be given by,

$$V_{o_1 rms} = \left[\frac{2}{2\pi} \int_{\frac{\pi-\delta}{2}}^{\frac{\pi+\delta}{2}} v_{dc}{}^2 \, d\,\omega t \right]^{1/2} = V_{dc} \sqrt{\frac{\delta}{\pi}}$$

The harmonic content can be reduced by using severed pulses in each half-cycle of output voltage. The frequency of reference signal sets the output frequency f_o , and the carrier frequency f_c determines the number of pulses per half-cycle P.

$$P = \frac{f_c}{2\, f_o}$$

As M varies from 0 to 1, δ varies from 0 to π / P and the output voltage from 0 to v_{dc} .

$$V_{o,rms} = \left[\frac{2P}{2\pi} \int_{\frac{\pi}{p}-\delta}^{(\frac{\pi}{p}+\delta)} v_{dc}{}^2 d(\omega t) \right]^{\frac{1}{2}} = Vdc\sqrt{p\frac{\delta}{\pi}}$$

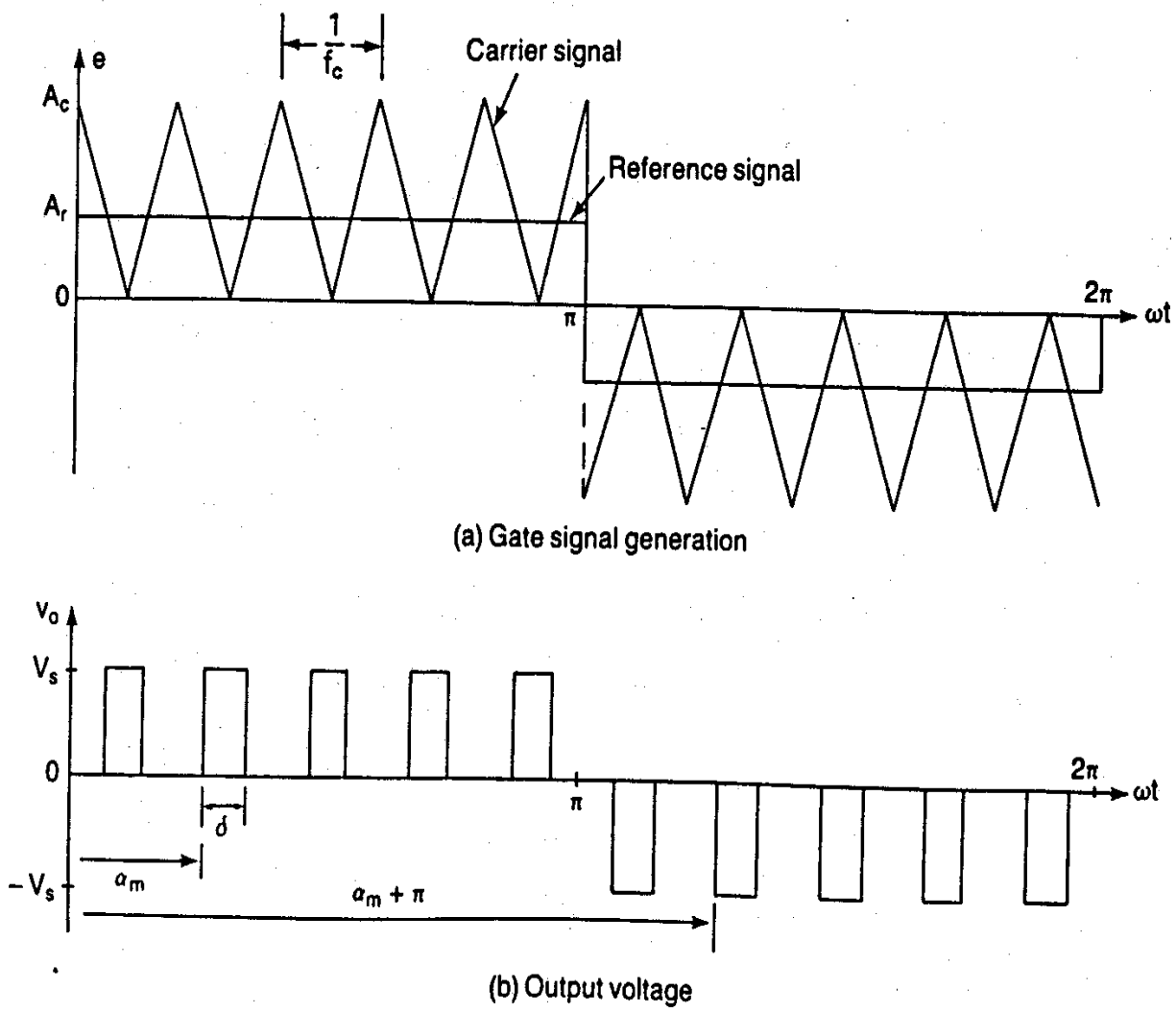


Fig.14.8

Since the waveform is half-wave symmetry, the Fourier series expansion for the output voltage is

$$V_o(t) = \sum_{n=1,3,5.....}^{\infty} B_n \sin n\omega t \,;\, A_0 = A_n = 0$$

15. THREE-PHASE INVERTERS

15.1 Three-phase inverters are normally used for high-power applications. Three single-phase parallel, inverters can be connected in parallel as shown in Fig.15.1 to form the configuration of a three-phase inverter.

The gating signals of single-phase inverters should be advanced or delayed by 120^0 with respect to each other in order to obtain three-phase balanced (fundamental) voltages. The transformer primary windings must be isolated from each other, while the secondary winding may be connected in wye or delta. The transformer secondary is normally connected in wye to eliminate triple harmonics (n = 3, 6, 9…) appearing on the output voltages.

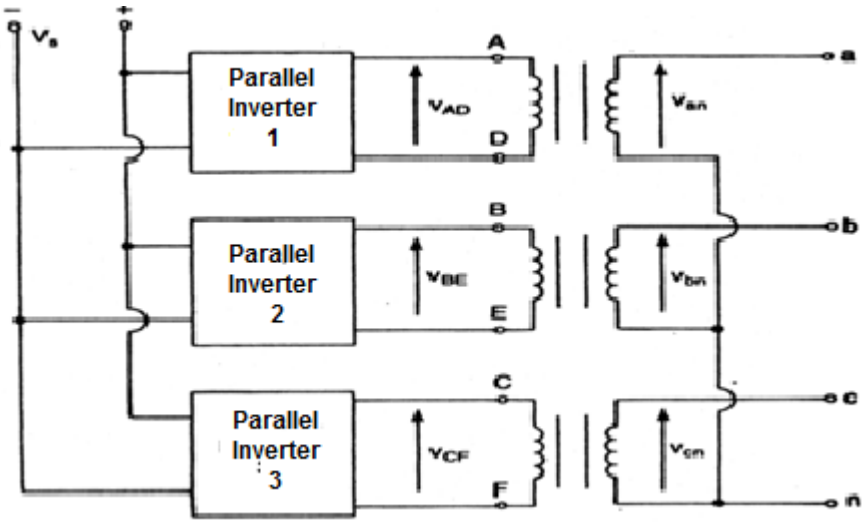


Fig. 15.1 Schematic diagram of a three-phase inverter constructed from three single-phase parallel inverters.

15.2. Three-phase Bridge inverter:

Three single-phase half- bridge inverters can be connected as shown in Fig.15.2 to form a configuration of three-phase inverter.

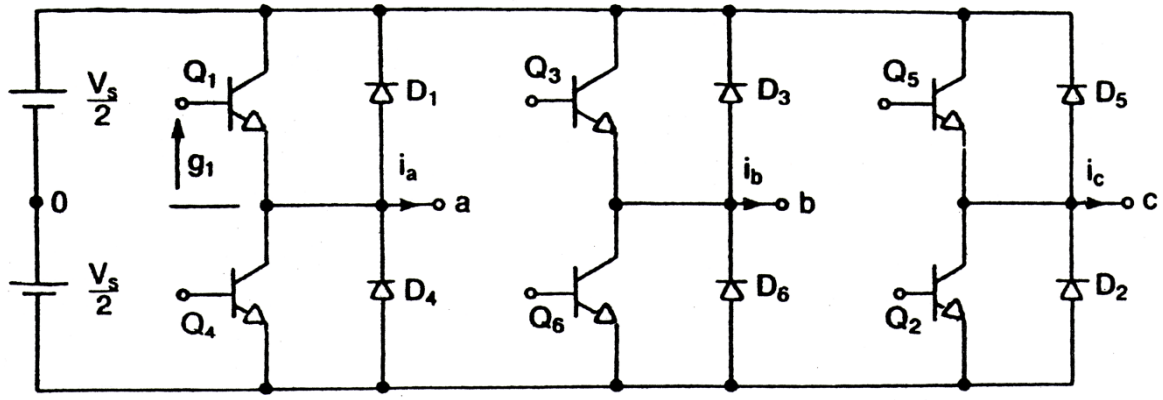


Fig.15.2

When transistor Q1 is switched on, terminal *a* is connected to the positive terminal of the dc input voltage. When transistor Q4 is switched on, terminal *a* is brought to the negative terminal of the dc source. There are two types of conductions depending on the control signal used:

1. 120^o conduction.

2.180^o conduction.

In this context we shall consider 120^o conduction mode only for clarity.

120 – Degree conduction:

In this type of control, each thyristor (or Transistor) conducts for 120^o. Only two thyristors remain "ON" at any instant of time. The per phase and line- to-line voltage waveforms are shown in Figs.15.3 and 15.4 respectively.

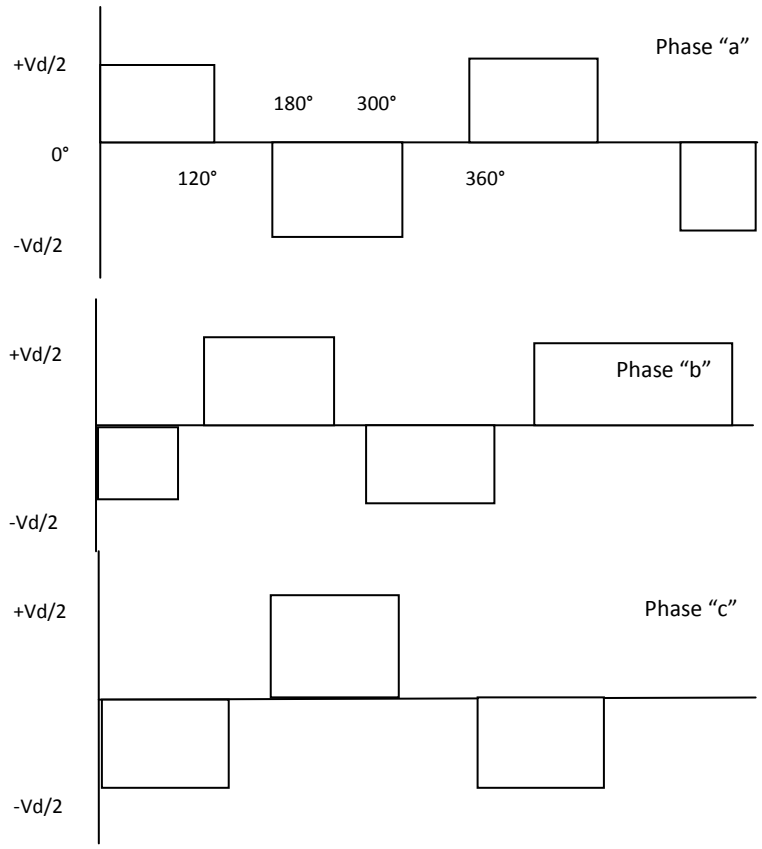


Fig.15.3 Per phase output voltage for three-phase bridge inverter

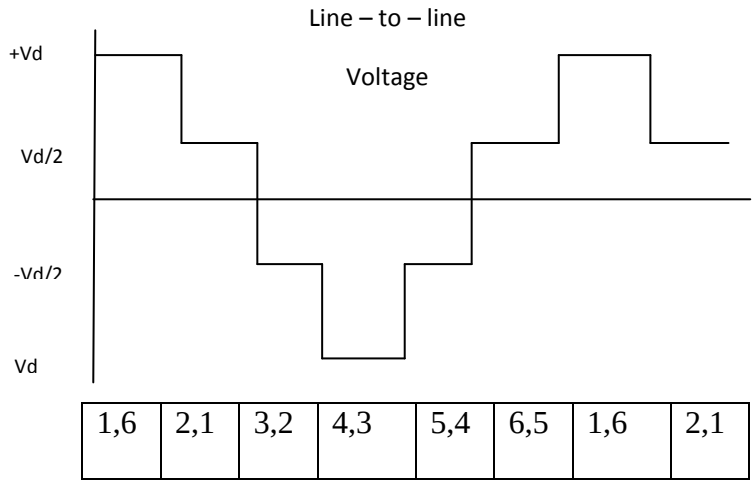


Fig.15.4 line to line voltage for three-phase Bridge inverter.

For star connected load, three modes for three-phase bridge inverter operation exist. In each case, the effective resistance across the source is 2R as shown 15.5.

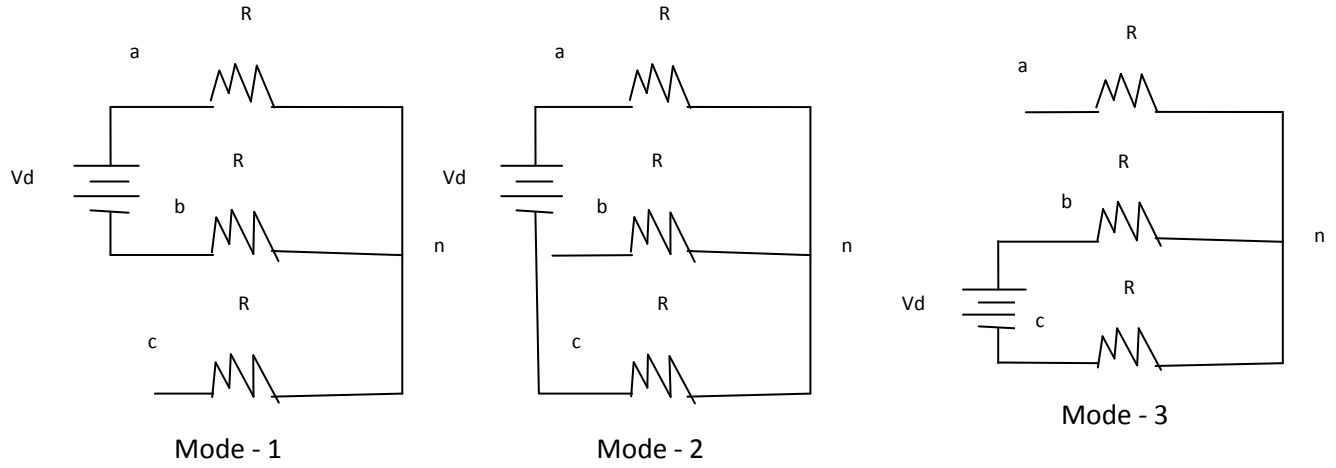


Fig.15.5 Modes for the three-phase bridge inverter.

For these three modes of operation:

Mode 1: Thyristors 1 & 6 conduct $0 \leq wt \leq 60°$

$$V_{an} = \frac{Vd}{2}, \qquad V_{bn} = -\frac{Vd}{2}, \qquad V_{cn} = 0$$

Mode 2: Thyristors 1 & 2 conduct $60° \leq wt \leq 120°$

$$Van = +\frac{Vd}{2}, \quad V_{bn} = 0, \qquad V_{cn} = -\frac{Vd}{2}$$

Mode 3: Thyristors 2 & 3 conduct $120° \leq wt \leq 180°$

$$V_{an} = 0, \quad V_{bn} = \frac{Vd}{2}, \quad V_{cn} = \frac{-Vd}{2}$$

- The line – to – neutral voltages can be expressed in Fourier series as:

$$V_{an}(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2Vd}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t + \frac{\pi}{6}\right)$$

$$V_{bn}(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2Vd}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t - \frac{\pi}{2}\right)$$

$$V_{cn}(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2Vd}{n\pi} \cos\frac{n\pi}{6} \sin n\left(\omega t - \frac{7\pi}{6}\right)$$

The peak amplitude of the fundamental component (n=1): for phase a, b or c is:

$$V_{1P} = \frac{2Vd}{\pi} \cos\frac{\pi}{6} = \frac{Vd}{\pi}\sqrt{3}$$

The r.m.s value of the fundamental component is

$$V_{1r.m.s} = \frac{V_{1P}}{\sqrt{2}} = \frac{Vd\sqrt{3}}{\sqrt{2}\pi}$$

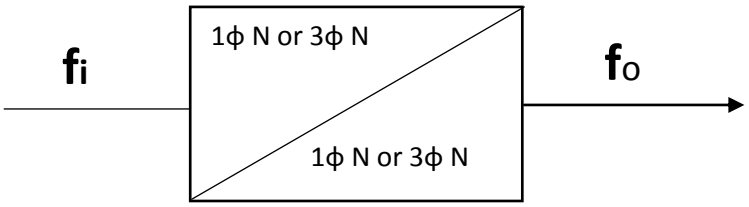
- Note: line – to – line voltages for star – connected load can by found as:

$$V_{L-L} = \sqrt{3}\, V_{phase}$$

16. Cycloconverters

A cycloconverter is a direct-frequency changer that converts **ac** power at one frequency to **ac** power at another frequency by **ac-ac** conversion.

Cycloconverters are used in high power applications driving synchronous and induction motor. They are usually phase controlled and they use thyristors due to their ease of natural (phase) commutation.



Cycloconverter Fig.1

16.1 single-phase cycloconverter:

To understand the principle of operation of cycloconverter, the single-phase to single-phase cycloconverter (Fig.2) should be studied first. This converter consists of back to ack connection of two full-wave bridge rectifier circuits. Fig.3 shows the operate waveforms of this cycloconverter with resistive load.

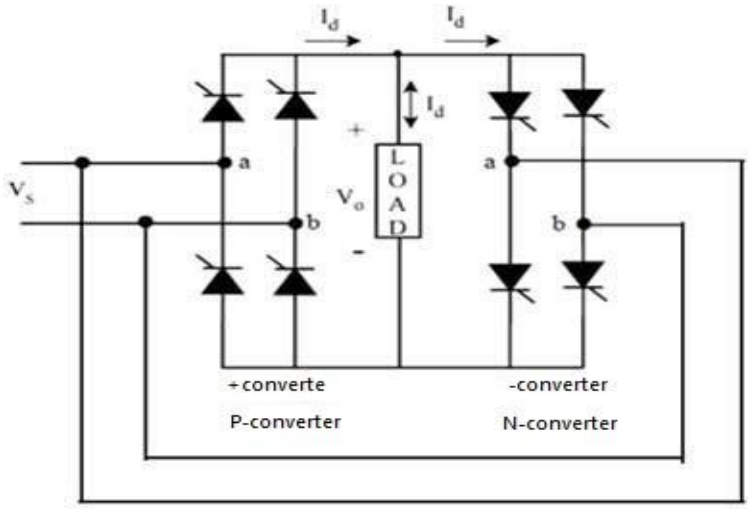


Fig.2 single-phase to single-phase cycloconverter

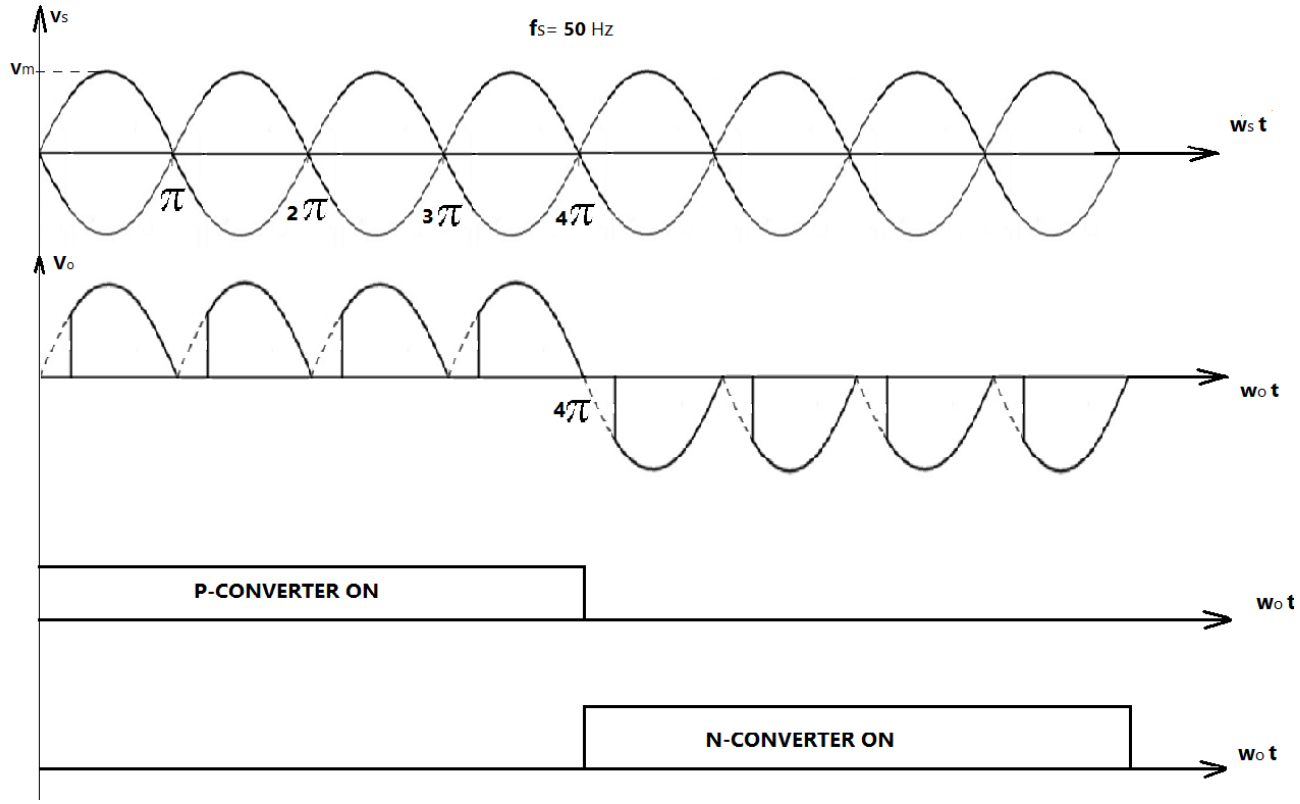


Fig.3 waveforms with resistive load.

If input voltage is

$v_s = V_m \sin w_s t$

The r.m.s output voltage can be found from:

$$V_o = \left\{\frac{8}{8\pi}\int_{\alpha p}^{\pi}\left(\sqrt{2}\,V_s \sin w_s t\right)^2 dwt\right\}^{\frac{1}{2}}$$

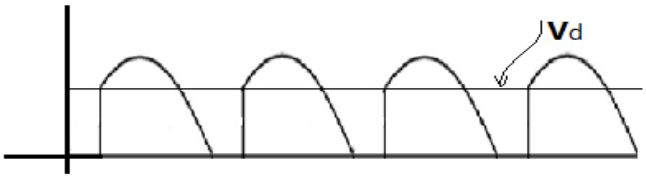
$$= \left\{\frac{16{V_s}^2}{16\pi}\int_{\alpha p}^{\pi}(1-\cos 2w_s t)\,dwt\right\}^{\frac{1}{2}}$$

$$= V_s\left[\frac{1}{\pi}\left(\pi - \alpha p + \frac{\sin 2\alpha p}{2}\right)\right]^{\frac{1}{2}}$$ (Not depend on f_o)

- When p-converter operating alone, the output voltage is positive half-cycles, with average output voltage of

$V_{d1} = \frac{2V_m}{\pi}\cos\alpha$

step down converter



The output frequency $f_o = \frac{1}{T_o} = \frac{1}{4}f_s$. Also, when N-converter operating alone, the output voltage is negative with average value: $V_{d2} = -V_{d1}$

Example: The input voltage to the cycloconverter in Fig.2 is 120 v (r.m.s), 50Hz. The load resister is 5Ω and the inductance is L=40 mH. The frequency of the output voltage is 25Hz. If the converters are operated as semiconverters such that

$0 \le \alpha \le \pi$ the delay angle is $\alpha p = \frac{2\pi}{3}$, determine:

(a) The rms value of the output voltage V_o.
(b) The rms value of the load current I_o.
(c) The input power factor.

Solution:

(a) The rms value of the output voltage:

$$V_o = V_s\left[\frac{1}{\pi}\left(\pi - \alpha p + \frac{\sin 2\alpha p}{2}\right)\right]^{\frac{1}{2}}$$

$$= 120\left[\frac{1}{\pi}\left(\pi - \frac{2\pi}{3} + \frac{\sin\frac{4\pi}{3}}{2}\right)\right]^{\frac{1}{2}}$$

(b) $Z = \sqrt{R^2 + (w_o L)^2} = \sqrt{(5)^2 + (2\pi * 25 * 40 * 10^{-3})^2} = 8.02$

$$\theta = \tan^{-1}\frac{6.28}{5} = 51.4^{\circ}.$$

The r.m.s load current is $I_o = \frac{V_o}{Z} = \frac{53}{8.02} = 6.6\ A$

(c) The input PF:

R.M.S input current $I_s = I_o = 6.6\ A,$

The VA rating is $VA = I_s V_s = 6.6 * 120 = 792\ VA = P_{in}$

The output power $P_o = V_o I_o \cos\theta = 53 * 6.6 * \cos 51.4$

$$= 218.23\ W$$

$$\therefore PF = \frac{P_o(W)}{P_{in}(VA)} = \frac{218.23}{792} = 0.2755\ (lagging).$$

16.2 Three-phase to single-phase cycloconverter:

The 3-phase to single-phase cycloconverter consists of two 3-phase half-wave controlled rectifiers connected as shown in **Fig.4** . One converter is for positive group and the other for the negative group.

- When p-converter operating alone, positive voltage (k) appears across the load, and when N-converter operating alone, negative voltage (L) appears across the load.

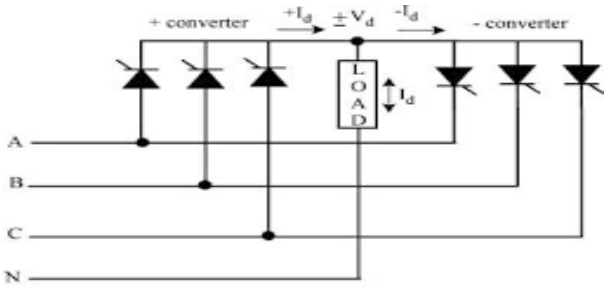


Fig. 4 3ϕ-1ϕ half-wave cycloconverter

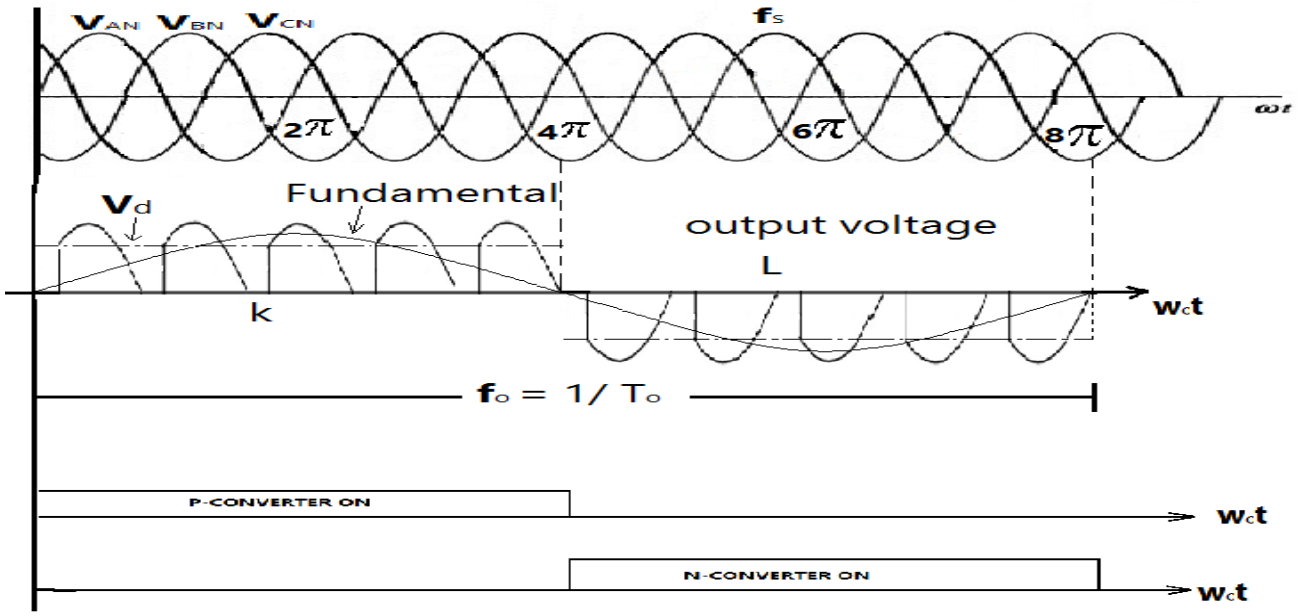


Fig.5 Waveform for resistive load

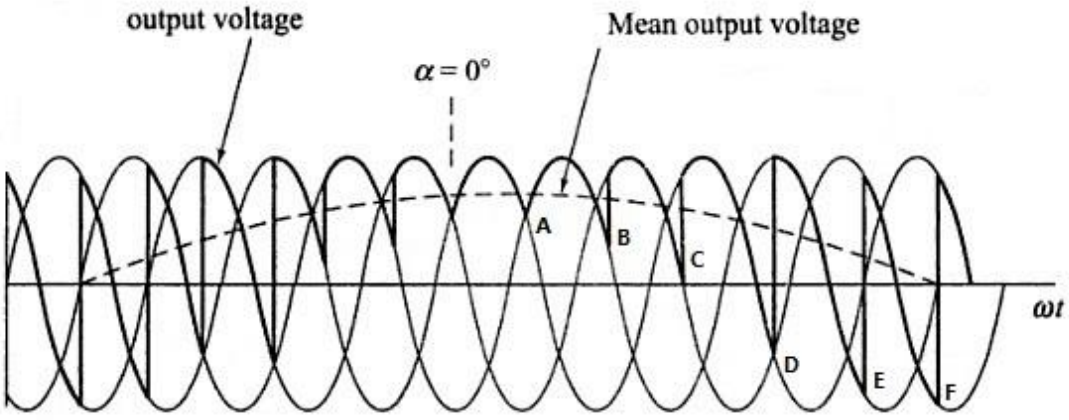
- For $\alpha \neq 0$

  For the positive group converter:

  At point "A" , $\alpha = 0$ $\quad V_d = \frac{3\sqrt{3}}{2\pi} V_m = V_{do}.$

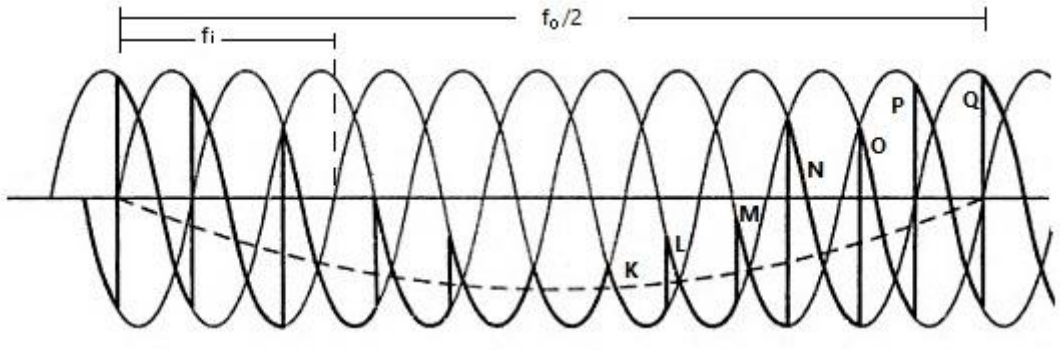
  At point "B" , $\alpha > 0$ $\quad V_d = V_{do} \cos \alpha$

  At point "F" , $\alpha = \frac{\pi}{2}$ $\quad V_d = 0$



Positive group converter operating

Negative group converter is idling



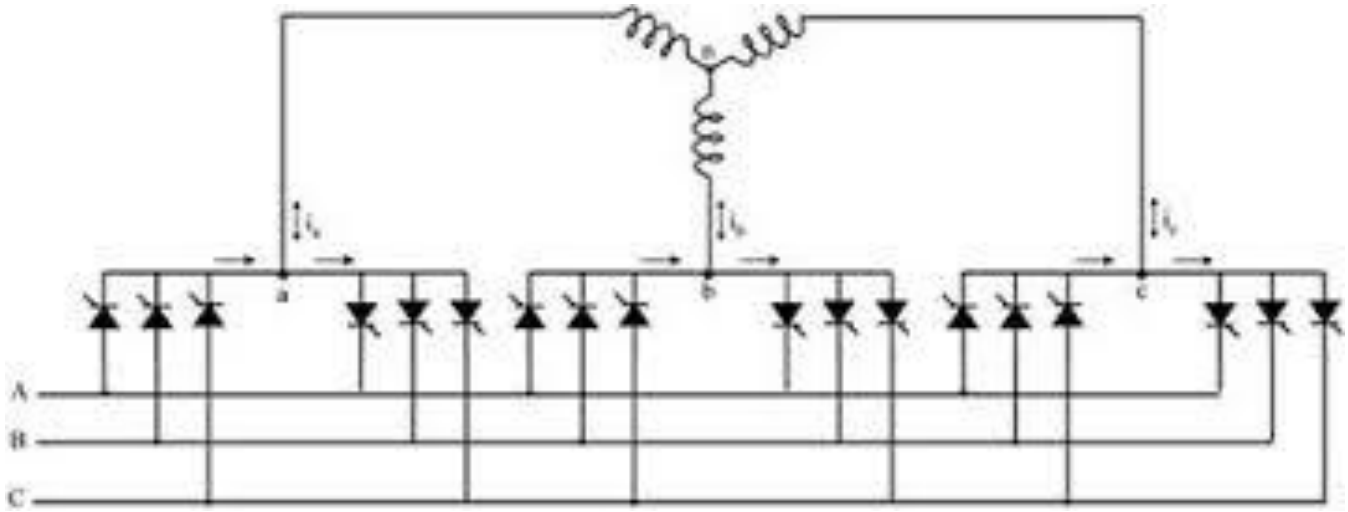
Negative group converter operating
Positive group converter is idling

At point "K" "L" "M" "N" "O" "P" and "Q"
$\alpha > 90°$, V_d is negative [inverting mode]

- Change over from positive group converter to negative group converter should be done at points "F" and "Q" where the output of the two converter are zero to avoid short circuit. Then the condition that $\alpha p + \alpha_N = \pi$ should be met.

16.3 Three-phase to three-phase cycloconverter:

- If the output of 3-phase to single-phase converters of the same kind are connected in $Y\ or\ \Delta$ and if the output voltages are $\frac{2\pi}{3}$ radians phase shifted from each other, the resulting converter is a three-phase to three-phase cycloconverter as shown in Fig.5



- Another type of 3Ø

to 3Ø cycloconverter is obtained by using Full-wave bridge rectifier as shown in Fig.6. This type of frequency changer is using 36 theyristor. The output frequency f_o is less than the input f_i .

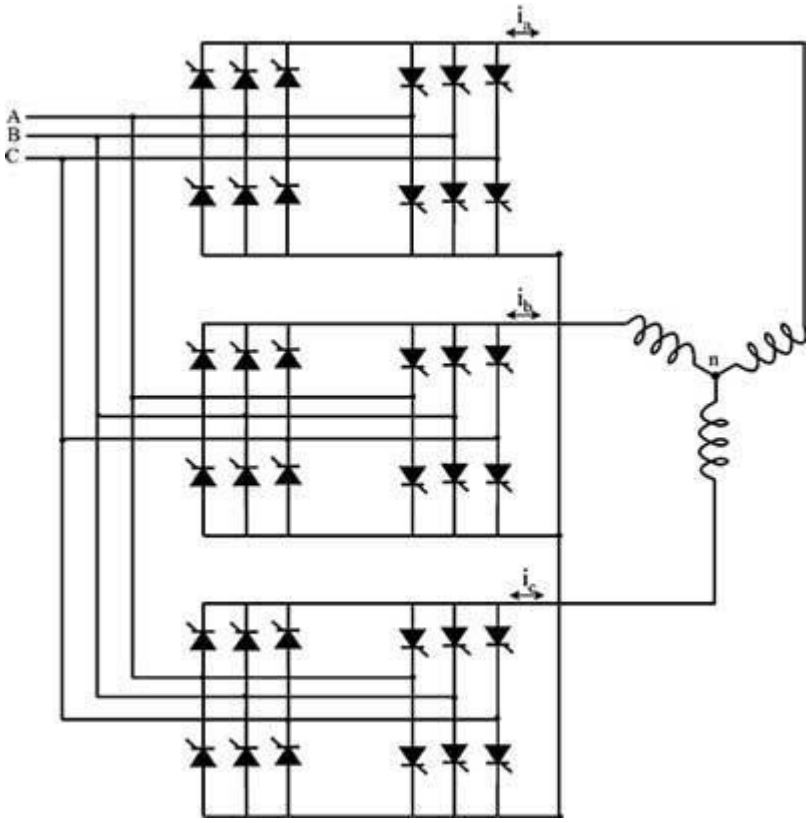


Fig.6 3Ø to 3Ø cycloconverter